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CALCULATING CUSHION THICKNESS BY ANALYSIS OF STRESS-STRAIN CURVES

R. E. JONES
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FOREST PRODUCTS LABORATORY
UNITED STATES DEPARTMENT OF AGRICULTURE

JANUARY 1954

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CALCULATING CUSHION THICKNESS BY ANALYSIS OF STRESS-STRAIN CURVES

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United States Department of Agriculture*

January 1954

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Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

FOREWORD

This report was prepared by the United States Forest Products Laboratory under USAF Contract No. AF 33(038)51-4065. The contract was initiated under Research and Development Order No. 618-11(A-E), "Improvement of Packaging Procedures", and was administered under the direction of the Materials Laboratory, Directorate of Research, Wright Air Development Center, with Mr. S. M. Birnbaum acting as project engineer.

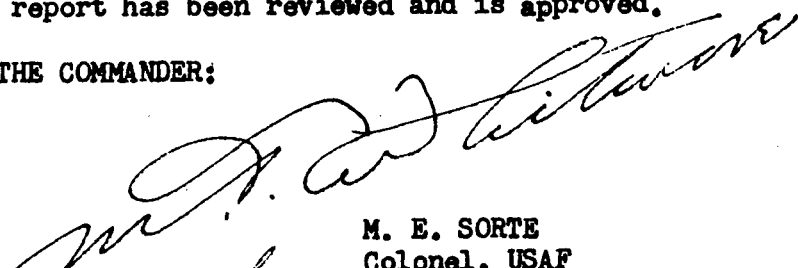
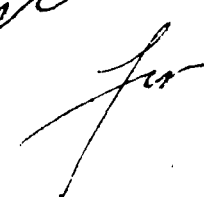
ABSTRACT

A method for designing package cushioning to protect an article against shock is developed from physical and mathematical concepts. Design curves for calculating cushion thickness are included for many currently available cushioning materials. Methods for selecting the proper cushion are discussed.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

M. E. SORTE
Colonel, USAF
Chief, Materials Laboratory
Directorate of Research

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INTRODUCTION

The U. S. Forest Products Laboratory in cooperation with the Packaging Branch (WCRTG), Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, has undertaken a project to determine cushion thickness design for protection of a packaged article against damage due to shock during handling operations. Specifically, the purposes of this report are:

- (1) To present a method for the calculation of cushion thickness for a given package design problem.
- (2) To present a method for obtaining the necessary cushion thickness design curves.
- (3) To provide design curves for some of the currently available cushioning materials.

The general principles on which this report is based were contained in a Laboratory memorandum report which was transmitted to our cooperator on December 30, 1952, as a proposed method for the design of cushions. The collection of data which could be analyzed by this method was subsequently approved.

Since 1944, a number of significant contributions concerning the design of package cushioning have been published. The design methods proposed in many of these studies (1)(2)(3)(4)(5)(6)(7)* possess certain similarities to each other as well as to the method forming the basis of this present work. That is, each of these works is based on the same

*Numbers in parentheses refer to references in the bibliography.

type of experimental data; each makes use of the same physical model for a complete pack; each assumes the existence of a single number which can be used to specify the fragility of an article; and, each is set up to solve essentially the same problem, that of calculating cushion thickness to protect a given article against damage due to a specific height of drop during handling operations. Brief abstracts of the portions of these investigations that are most closely related to the methods of analysis employed in this study are given in the bibliography.

CRITERIA FOR HANDLING AND FRAGILITY

In order to design a package rationally to protect a given article against damage during handling operations, criteria of fragility and a level of performance in rough handling must be established for the article. The complexity of these problems can be appreciated by listing some of the variables.

Level of Performance

The determination of the hazards encountered by a package during handling operations is, in the final analysis, a statistical problem. If the statistical distribution of forces encountered in handling operations is known, the package can be designed for the successful transport of any desired percentage of the total shipment. In packaging for commercial shipment it might be economically feasible to permit damage to a limited percentage of the goods shipped; whereas, for strategic military materials,

it would be necessary to design for nearly 100 percent successful shipment. In this analysis, such factors as height of drop, type of surface, probability of a flat drop, and orientation of container at impact (edge, corner or flat drop) might be considered. Since information is not available on all these factors, the height of drop is used herein as a basis for specifying the level of performance in handling operations. Although a 30-inch drop is often taken as the assumed height of drop because of its relation to the height of a man's hands from the floor when standing, the methods used herein may be applied to any height of drop.

Article Fragility

As is the case with criteria for rough handling, numerous factors could be used to establish criteria of fragility. Variation in materials and the fatiguing effect of a series of equal or unequal shocks could again lead to a statistical approach to the problem.

Neglecting these factors, the effect of a certain type of shock on an article could be determined by applying the complete acceleration versus time curve of the shock to the article. Such an analysis would, as pointed out by J. P. Walsh of the Naval Research Laboratories, at the Navy's 20th Shock and Vibration Symposium, be prohibitively expensive and time consuming for most applications.

To simplify this problem, the article is assumed to be rigid and only the peak acceleration is used in this report as a criterion of fragility. It is obvious that this quantity alone is not completely sufficient. For example, an article given a constant acceleration in a centrifuge would

give no indication of collision damage, such as could occur in electronic equipment where a shock-mounted tube tipped far enough during shock to strike an object, such as a transformer. Thus, the fragility must vary with the pulse shape; that is, with the rate at which the acceleration is applied. This would lead to the problem of selecting the pulse shapes for which an article must be tested. A practical solution to this problem has been suggested by Marvin Masel(5). His method consists essentially of determining fragilities by dropping an article on various cushions until damage occurs. The advantage here is that, since the article is tested under a situation similar to actual packaging conditions, the pulse shape need not be considered unless the fragility expressed by the maximum acceleration varies sharply as a function of pulse shape.

While the use of maximum acceleration as a criterion for fragility is an oversimplification, it is thought that its use does lead to improved design. For package cushioning it also provides a simple basis for comparison of materials as to their effectiveness in limiting peak acceleration.

Another reason for selecting the maximum acceleration is that it can be related to the maximum force for a rigid body through Newton's second law of motion, thus

$$F_m = ma_m \quad (1)$$

where F_m = maximum force in pounds

m = mass in slugs

a_m = maximum acceleration in feet per second per second

Since,

$$W = mg \quad (2)$$

where W = weight in pounds

g = acceleration due to gravity or 32.2 feet per second per second

we obtain by combining equations (1) and (2)

$$F_m = \frac{Wa_m}{g} \quad (3)$$

To facilitate the use of acceleration in this formula, the term g -value is defined as the ratio of an acceleration to the acceleration due to gravity, thus

$$G = \frac{a}{g} \quad (4)$$

where G = g -value, a dimensionless ratio

a = acceleration in feet per second per second

g = acceleration due to gravity or 32.2 feet per second per second

The maximum g -value and acceleration a given article can sustain without being damaged are given by G_m and a_m , respectively, where

$$G_m = \frac{a_m}{g} \quad (5)$$

Thus, if the maximum acceleration a given article can withstand is 3,220 feet per second per second or 100 g , the maximum g -value would be 100.

Substituting equation (5) into equation (3), we obtain

$$F_m = WG_m \quad (6)^a$$

^aEquation (6) could be modified to take into account the weight of the article. For a falling body, a vertical force acting upward on the article would equal the sum of the inertial force and weight of the article, thus $F_m = WG_m + W$ or $F_m = W(G_m + 1)$. Since the maximum g -values for articles are generally assumed to be greater than 10, equation (6) can be used to a good approximation.

Therefore, a rigid article having a maximum g-value of 50 and weighing 2 pounds could sustain without damage forces up to 100 pounds.

If the maximum force an article can sustain dynamically without damage is known, it can be used just as effectively as maximum g-value or acceleration. Acceleration is used here since the quantities displacement, velocity, and acceleration have been generally used in shock and vibration studies. The use of g-value and acceleration also emphasizes the dynamic characteristics of the problem.

MECHANICAL MODEL OF CUSHIONED PACK

A cushioned pack, as shown in figure 1, consists essentially of (a) an article, often placed in a chipboard or fiberboard container; (b) a cushioning material; and, (c) an outer container. In order to analyze a cushioned pack by simple physical laws, it is necessary to use an idealized mechanical system representing a package during a drop, such as that shown in figure 2.

This model has the following characteristics:

(a) The article, assumed to be rigid, is represented by the weight, W .

(b) The cushion, assumed to be massless, is represented by a spring with spring constant, K .

(c) The outer container and floor are assumed to be rigid.

(d) Only flat drops are considered because edge or corner drops would be difficult to analyze mathematically, and it is assumed that the flat drop will be the most severe on the article.

Accelerations During a Drop Test

When a cushioned pack is dropped from a height, h , (fig. 2A) it receives a constant acceleration of 1 g until the instant the container strikes the floor (fig. 2B). At this instant, the container stops, and the article continues to fall a distance, s , as shown in figure 2C. The article will stop falling when the cushion has absorbed or stored all the potential energy possessed by the article at a height, h . The acceleration (negative) experienced by the article will depend on how quickly it is stopped and will in general be much greater than 1 g . The maximum acceleration will be encountered at the maximum displacement of the cushion when the velocity of the article is zero.

TYPES OF CUSHIONS

Since, for a given height of drop and a given article, the maximum acceleration depends on the force-displacement curve of the cushion, the general shapes of these curves are used to identify cushion types. These curves for three types of cushioning materials are shown in figure 3. An experimental procedure for obtaining these curves is given on page 14. Cushions exist that possess nearly all the characteristics of a cushion with ideal elasticity (fig. 3A) but they generally have undesirable recovery characteristics. An example of this is plastic-foam cushioning material. Linear elasticity (fig. 3B) is characteristic of steel springs. The curve shown in figure 3C is typical of many commonly used cushioning materials and can be fit to tangent, cubic or logarithmic functions fairly

well depending on the particular material being considered. Since it is unnecessary in the method of design used in this article to fit the force-displacement curves of cushions to any specific mathematical function, design for these various types will be included under anomalous elasticity.

Cushions with Ideal Elasticity

When an article is packed with a properly designed cushion with ideal elasticity, it receives a constant acceleration equal to the maximum permissible acceleration for the article from the time the container strikes the floor until the article is stopped. Thus, the article is stopped in the minimum distance without exceeding its maximum g-value. The equation for a cushion with ideal elasticity (fig. 3A) is

$$F = K' \quad (7)$$

where

K' = spring constant in pounds for a cushion with ideal elasticity

F = force exerted by the cushion on the article in pounds

From the law of conservation of energy, the potential energy of the article at height, h , must equal the energy stored or absorbed by the cushion when the article comes to rest after impact, thus

$$Wh = \int_0^{T'} F \, dS \quad (8)$$

where S = displacement of cushion

T' = maximum displacement of a cushion with ideal elasticity

F = force which, for a cushion with ideal elasticity, is also the maximum force, F_m , in pounds

Integrating equation (8) and solving for T' we obtain

$$T' = \frac{Wh}{F_m} \quad (9)$$

Substituting equation (6) for F_m in equation (9) gives

$$T' = \frac{h}{G_m} \quad (10)$$

Thus, equation (9) gives the minimum stopping distance or cushion thickness required for a given cushion design problem. Note that for $G_m = 1$, the article must fall a distance equal to the height of drop after the container stops and that the thickness, T' , does not include any space for the compressed cushion at the maximum displacement.

Cushions with Linear Elasticity

When an article is cushioned with a cushioning material having linear elasticity, it is subject to an increasing acceleration from the time the container strikes the floor until the article is stopped. If the cushion is properly designed and the height of drop is the same as that for which the cushion is designed, the maximum acceleration which will occur just as the article stops will equal the maximum permissible acceleration for the article. The equation for a cushion with linear type elasticity (fig. 3B) is

$$F = K''S \quad (11)$$

where K'' = spring constant in pounds per inch of a cushion with linear elasticity, the energy equation (see equation 8) becomes

$$Wh = \int_0^{T''} F \, dS \quad (12)$$

where T'' = maximum displacement of a cushion with linear elasticity.

Substituting equation (11) for F in equation (12) and integrating, we obtain

$$Wh = \frac{K''T''^2}{2} \quad (13)$$

Using equation (11), we can write $F_m = K''T''$ (14)

From equations (6), (13), and (14) we obtained

$$T'' = \frac{2h}{G_m} \quad (15)$$

Cushions with Anomalous Elasticity

In order that a single curve can be used to represent the elastic property of a given cushion for any area or thickness, the force-displacement data is converted to stress-strain curves. The stress is obtained by dividing the force by the area of the cushion; thus,

$$f = \frac{F}{A} \quad (16)$$

where f = stress in pounds per square inch

F = force applied to cushion in pounds

A = area of cushion in square inches

The strain, a dimensionless quantity, is obtained by dividing the displacement by the original thickness of the cushion; thus,

$$s = \frac{S}{T} \quad (17)$$

where s = strain in inch per inch

S = displacement or change in cushion thickness in inches

T = measured thickness of cushion in inches

This is the usual definition of strain provided the strain is constant throughout the thickness of the cushion. The modulus of elasticity is defined as the slope of the stress-strain curve at a given strain. For anomalous elasticity, the modulus of elasticity is not a constant as was the spring constant for cushions with ideal and linear elasticity. A typical stress-strain curve for anomalous elasticity is shown in figure 4.

Since we are using a curve of force per unit area versus displacement per unit thickness, the energy absorbed or stored per unit area per unit thickness of cushion can be equated to the integral of the stress-strain curve (see equations 8 and 24); thus,

$$\frac{Wh}{AT} = \int_0^{s_m} f \, ds \quad (18)$$

where s_m = maximum strain

Because stress is an unknown function of strain, the $\int f \, ds$ is also unknown. However, the $\int f \, ds$ for each value of strain can be obtained graphically from the area under the stress-strain curve. This is shown by the dashed curve in figure 4, where energy per unit volume is plotted against strain for the stress-strain curve shown.

For simplicity in notation, let e_m represent the area under the stress-strain curve from 0 to s_m ; thus,

$$e_m = \int_0^{s_m} f \, ds \quad (19)$$

Combining equations (18) and (19) and solving for T yields

$$T = \frac{Wh}{Ae_m} \quad (20)$$

From equations (6) and (16) the maximum stress is obtained as

$$f_m = \frac{WG_m}{A} \quad (21)$$

Equations (20) and (21) combined with the curves in figure 4 provide sufficient information to calculate cushion thickness.

Thickness Efficiency

The thickness efficiency of a cushion is defined in this report by

$$\text{Efficiency} = \frac{T'}{T} \times 100 \text{ percent} \quad (22)$$

where T = measured thickness of cushion

T' = thickness of a cushion with ideal elasticity for the same cushion design problem

From equations (10), (15) and (22) it can be shown that the maximum thickness efficiency of a linear cushion, T'/T'' , will be 50 percent. For most cushions, the thickness efficiency will be much less than 50 percent. Although many cushions are nearly linear up to a strain of 0.6, the efficiency is low because the measured thickness of the cushion, T , includes the space required by the cushion at maximum displacement.

Illustration

As an example of design by this method, find the least thickness of cushioning material No. 1 (fig. 4) for the following conditions:

Height of drop, $h = 30$ inches

Weight of article, $W = 2.5$ pounds

g -factor of article, $G_m = 32$

Bearing area of cushion, $A = 40$ square inches

Then the stress (equation 21) will be $f_m = \frac{WG_m}{A} = \frac{2.5 \text{ lb.} \times 32}{40 \text{ in.}^2} = \frac{2 \text{ lb.}}{\text{in.}^2}$

This stress corresponds to a strain of 0.8. The energy absorbed per unit volume of cushion at this strain is 0.31 inch-pounds per cubic inch.

Therefore, the required thickness (equation 20) will be

$$T = \frac{Wh}{Ae_m} = \frac{2.5 \text{ lb.} \times 30 \text{ in.}}{40 \text{ in.}^2 \times 0.31 \frac{\text{in.-lb.}}{\text{in.}^3}} = 6.1 \text{ in.}$$

The maximum displacement (equation 17) will be

$$S = sT = 0.8 \times 6.1 \text{ in.} = 4.9 \text{ in.}$$

An ideal cushion (equation 10) would require only

$$T' = \frac{h}{G_m} = \frac{30 \text{ in.}}{32} = 0.94 \text{ in.}$$

Therefore, the thickness efficiency (equation 22) is

$$\text{Efficiency} = \frac{T'}{T} \times 100\% = \frac{0.94 \text{ in.}}{6.1 \text{ in.}} \times 100\% = 15\%$$

TEST PROCEDURE FOR OBTAINING STATIC FORCE-DISPLACEMENT DATA

Materials Tested

Cushioning materials tested in this study were submitted by manufacturers as standard production products late in 1952 or in 1953. Many of the materials were supplied in different densities and thicknesses. Of the 60 materials tested, 9 were submitted as a second thickness of a specific cushion. A list of the materials tested are given in table 2 of Appendix I.

Conditioning

Three samples of each material were prepared for test. The samples for materials 16, 17, 18, 35, 36, and 37 measured approximately 5 by 5 inches; all other samples measured approximately 10 by 10 inches. All samples were cut on a bandsaw except materials 14 and 15 which were cut with a paper knife. The samples were conditioned at 75° F. and 64 percent relative humidity until they had acquired equilibrium moisture content. The samples were considered to have reached equilibrium moisture content when their weight did not change by more than 2 percent over a period of 24 hours.

Length and Width Measurements

The length and width measurements were determined by using the measuring jig illustrated in figure 5. The test specimen was placed on a plane surface with one edge adjacent to a stationary block. A movable block was then placed firmly against the opposite edge of the specimen, and the distance between the two blocks along a line equidistant from the free edges of the specimen was measured to the nearest hundredth inch. The specimen

was turned bottom side up, and a second reading was determined. The average of the readings was considered the length dimension of the specimen. This measurement procedure was repeated to obtain the width dimension of the specimen.

Thickness Measurement

The thickness of a specimen was determined by using the jig shown in figure 6. The specimen was placed on a plane surface and covered with a glass platen of such size and weight that the entire surface of the specimen was stressed to 0.01 pound per square inch. A dial indicator attached to a support was used to measure the thickness of the specimen to the nearest thousandth of an inch at its midpoint. This thickness will be called the measured thickness in this report.

Static Force-displacement Test

The force-displacement tests were run on a universal testing machine. Figure 7 illustrates a cushion between the flat platens before testing. The platen separation, before test, was adjusted to the measured thickness. This condition was assumed to be zero force and zero displacement. The distance between the platens was then decreased at the rate of approximately 0.3 inch per minute per inch of thickness. For most cushions the maximum force was approximately 4 times the value of force corresponding to the point of greatest curvature on the force-displacement curve. The force was recorded manually at increments of displacement equal to approximately 10 percent of the measured thickness.

Moisture Content and Density Determination

After testing, each specimen was conditioned in an oven at 100° C. until the weight did not change by more than 2 percent over a period of 24 hours. The density (ovendry) was expressed in pounds per cubic foot, and the moisture content in percent of ovendry weight. These quantities are listed in table 2 of Appendix I.

COMPARISON OF STRESS-STRAIN CURVES

Calculation of Stress and Strain

The forces for each of the three specimens at equal displacements were averaged and divided by the average area for the specimens to obtain the stress. The displacement at which the forces were averaged was divided by the average measured thickness for the three specimens to obtain the corresponding strain. Stress-strain curves were obtained by following this procedure for a series of points. In order to show clearly the entire range of the data obtained, it was necessary to use two stress axes for many of the cushioning materials. Whenever 2 coordinate systems were used, the ordinates were related to each other by a factor of 10. Stress-strain curves for 51 cushion materials are given in Appendix II.

Quality Control

The stress-strain curves given in the appendix represent in general an average of three specimens of the same nominal thickness and density. In order to show the spread in data for the three specimens, separate stress-strain curves for each specimen of material No. 2 were plotted as shown in figure 8. Since these specimens were cut from a single shipment of

material it is possible that larger variations exist between different shipments of the same material, which are of the same nominal thickness and density.

Although 2 thicknesses were tested of 9 materials, separate stress-strain curves were not included for the second thickness. Figure 9 illustrates the average stress-strain curves of two specimens of the same material with the same nominal density but of different thicknesses. This variation was considered typical of the materials for which two thicknesses were tested.

The data obtained in this study were checked against manufacturers' published data which were available for 10 materials. In general, the variation was much greater than that shown in figures 8 and 9. This discrepancy may be due to factors such as changes in manufacturing process, differences in densities, method of testing, and number of specimens tested.

Density

For a specific kind of material, such as fiberglass or curled cattle hair, the stress-strain curves show that, as the density of the material increases, the stress increases at a given value of strain. Illustrations of this for a specific material are shown in figures 10 and 11. Some manufacturers indicate increasing densities by soft, medium and firm, while others use nominal densities such as 1, 3, and 5 pounds per cubic foot. Since no general correlation between densities of cushions using different raw materials is known to exist, it is not possible to describe the energy absorbing properties of cushions in general by density.

The densities for the series of materials submitted by some manufacturers varied only slightly. When the variations in the separate specimens for 2 such materials was greater than the variation in the average stress-strain curves at a strain of 0.6, only 1 curve is presented for the 2 materials. An example of this is the stress-strain curve given for materials 16, 17, and 18 in the appendix. For materials 44, 45, 48 and 49 the variation in the specimens was greater than the average curves for several materials so that a band was used rather than a single line to show the stress-strain curve.

Comparison of Materials

In order to show the range of materials tested and the similarities in stress-strain curves for various materials, a set of composite curves was plotted. Figure 12 shows the boundaries of the four plots given in figures 13, 14, 15 and 16. The use of four different stress axes was necessary to provide a large enough scale to separate most of the curves. For several materials, it was not feasible to give a separate curve. The circled numbers give the actual material plotted, whereas numbers in parentheses give materials whose stress-strain curves were quite similar. These curves illustrate that some series of similar materials cover a very small range (e.g. 19, 20, 21) as compared with others (e.g. 5, 6, 7, 8, 9).

ADDITIONAL FACTORS THAT INFLUENCE THE SHAPE OF THE STRESS-STRAIN CURVES

In addition to the type and density of the cushioning material, other factors, such as temperature, humidity, irregularities on the surface of the article or container, rate of displacement, and repeated loading, can influence the shape of the stress-strain curve.

For this particular project, data were obtained to show the influence of repeated loading. Rate of displacement was also considered since the rate at which the cushions were displaced during test was slow compared to the rate encountered in an actual drop, thus making the data valid only to the extent that static tests are equivalent to dynamic tests. No actual tests were made; however, previous work (3) showed that the agreement between the displacement predicted from static force-displacement data and the actual displacement measured during a drop of a weight on a cushion was good for the materials tested.

Repeated Loading

Since an article that is cushioned within a package and shipped is subjected to repeated rough-handling shocks before it is unpacked, it is logical to choose cushioning material on the basis of its performance characteristics after repeated loading. The following test procedure was used to illustrate the difference between cushion properties for initial and repeated loading.

Procedure

The initial stress-strain curve of a cushioning material was first examined to determine a value of strain where the stress increases rapidly for small increments of strain. This value was arbitrarily chosen as the strain for the first repeated-loading test.

Three conditioned samples, measured and weighed as described under procedure for initial-loading tests, were loaded and unloaded 10 times to the first arbitrarily chosen value of strain. The force-displacement data were recorded on the 9th and 10th loading. The thickness of the test specimen was measured 1 minute after the 9th and 10th unloadings. The force-displacement data on the 3 specimens for the 10th loading was averaged and transposed to stress (p.s.i.) and strain (inch per inch) as discussed previously. A stress-strain curve for the 10th loading was plotted on coordinate axis with the stress-strain curve for the initial loading (figs. 17 and 18).

A second value of strain 10 to 20 percent less than initial value was chosen, and a second group of 3 specimens was given the repeated-loading test. This procedure was repeated at subsequent values of strain that were approximately 10 to 20 percent less than the value of strain used on the preceding repeated loading. The maximum values of stress for the arbitrary values of strain on repeated-loading curves were joined to indicate the design loading stress-strain curve. The stress-strain curves for initial loading and design loading were transferred to a second coordinate system which had an energy per unit volume axis opposite the stress axis (figs. 19 and 20).

The areas under the initial-loading stress-strain curve, for intervals of strain of 0.10 inch per inch of thickness, were determined with a planimeter and plotted as energy per unit volume versus strain. The areas under the repeated-loading stress-strain curves for the arbitrarily chosen values of strains were also determined with a planimeter and plotted as energy per

unit volume versus strain on the same coordinate system (figs. 19 and 20). This curve sheet therefore illustrates stress-strain curves on initial and design loading and energy per unit volume-strain curves for initial and design loading.

Figures 17 and 18 represent data obtained from repeated loading of two different materials. In both figures, curves numbered (1) are the initial stress-strain curves which coincide with stress-strain curves for materials 2 and 9 given in the appendix. Curves numbered (2), (3), (4), and (5) represent data obtained on repeated-loading tests to arbitrarily chosen values of strain. Curve No. 6 is the design-load stress-strain curve, and it joins the maximum values of stress for repeated loading at arbitrarily chosen values of strain.

The values of set 1 minute after the 10th repeated unloading for arbitrarily chosen values of strain are illustrated on figures 17 and 18. Set in this report is defined as the ratio of the difference between the initial measured thickness and the measured thickness 1 minute after the 10th repeated loading to the initial thickness, times 100 percent.

$$\text{Set} = \frac{T - T_{10}}{T} \times 100 \text{ percent} \quad (23)$$

where T = initial measured thickness

T_{10} = measured thickness after the 10th repeated loading

Note: Although in the repeated-loading tests just described, the cushion was stressed to equal strains on each loading, this does not reproduce exactly what would happen from a series of drops from equal heights.

Repeated drops from a given height would cause a cushion to be stressed to equal energy absorbed per unit volume of cushion. This condition would be difficult to simulate in static tests, however, and since it is doubtful that in actual handling a package is dropped repeatedly from the same height, the procedure for repeated loading is recommended for the evaluation of cushioning materials.

Results

The results of the repeated-loading tests on materials 2 and 9 indicated that the stress-strain data for the 9th and 10th loadings for all strains tested were the same within limits of sample variations. The measured thicknesses of the samples had become constant within limits of sample variations by the 10th loading.

Figures 17 and 18 show that set is a function of strain. Figures 19 and 20 indicate that for materials tested there is more energy absorption capacity available on the initial loading than on the 10th repeated loading. Therefore, the selection of cushioning material and cushion thickness should be based on the design-loading stress-strain curves rather than the initial-loading stress-strain curves. For some materials, however, the initial- and design-loading stress-strain curves may be the same within the limits of the variations of test samples.

The ratio of the energy absorption capacity after repeated loading to that of initial loading at various strains is not necessarily dependent on the set. For example, the repeated-loading tests indicate that material No. 2 receives a greater set than material No. 9, but material No. 2 has smaller decrease of energy absorption capacity.

A resilient material may be defined as a material that has the same stress-strain curves for repeated loading as for initial loading. Some cushioning materials are resilient for small strains while others are resilient for strains above 50 percent. For a practical concept of resilient materials, tolerances could be chosen to allow for some differences between the initial loading and the 10th repeated loading.

SELECTING THE PROPER CUSHION

The choice of a cushion will be influenced by several factors, some of which are beyond the scope of this paper. For a given application many types of cushioning materials may be eliminated due to physical and chemical characteristics, such as dusting, pH, acidity, and mold and bacteria resistance. Others may be eliminated because of economic factors, such as cost, reusability, availability of the cushioning material and the basis for shipping charges (weight or cubage). Factors to consider in the choice of cushions relative to their energy absorption and peak acceleration are (1) the point of design on the stress-strain curve and (2) the thickness of cushion required.

Point of Design

The point of design on the stress-strain curve for a given material is fixed by the stress as determined through equation 20. For most cushioning materials, the modulus of elasticity increases as the strain increases. Thus, for a given increment in stress, the increase in energy absorption is much less at high strains than at low strains. Therefore, a small increase in the height of drop would cause a large increase in stress.

Also, for some types of cushions, large strains cause considerable set and loss in energy absorption. For these reasons, it is recommended that design at large strains be avoided. At other strains, the limiting factor will be the thickness of cushion required. A discussion of the factors involved in making the bearing area of the cushion different from the area of the article to change the point of design is contained on page 26.

Cushion Thickness

In order to calculate the cushion thickness, it is necessary to know the energy absorbed per unit volume of cushion for each value of strain. This was computed from the areas under the stress-strain curves which were obtained with a planimeter at increments in strain of 0.1. These curves are shown as dashed lines on the coordinate systems contained in the appendix.

If only a few cushioning materials are available to choose from, the cushion thickness may be calculated by the method shown on page 12 for each material. Once the thickness required of each material is known, account may be taken of the factors already given, such as cost.

A more systematic method of approach to the problem of cushion thickness is to plot curves of stress versus energy per unit volume as shown in figures 21 through 24. If it is assumed that the weight, height of drop, and bearing area are fixed for a specific problem, then the only remaining variable in the thickness formula (equation 20) is energy per unit volume. Therefore, at a given stress the thickness of cushion required is inversely proportional to the energy per unit volume; that is, the cushioning material with the largest energy per unit volume will require the least thickness.

Although the materials for which curves are shown in figures 21 through 24 cover a wide range of stress-strain curves (see figures 12 through 16), it can be shown that the choice of cushion is not critical for many problems as far as thickness is concerned. When the cushion design problem given on page 12 is solved for all the materials given in figures 21, 22, and 23, the results obtained are as summarized in table 1 on page 26.

For this illustration, the stress is 2 pounds per square inch. At this stress, the strain varies from 5 to 84 percent for the materials being considered. For low strains (materials Nos. 1 and 5), and for high strains (materials Nos. 7 and 8), the required thickness is much larger than for intermediate strains. If it is assumed that the uncertainty in package design is ± 10 percent due to factors, such as variation in cushioning materials, the use of nominal stocks of cushions, lack of information on handling conditions, and fragility of articles, then the variation in required thickness for materials 2, 3, 4, 6, 50, 51, 52 and 53 is not large.

Comparison of the curves in figures 21, 22 and 23 shows that at stresses between 2 and 4 pounds per square inch certain materials have a much lower energy absorption per unit volume than others. Figures 12 through 16 show that the stress-strain curves for these materials fall below (materials Nos. 1 and 5) or above (materials Nos. 7 and 8) the stress-strain curves for other materials (2, 3, 4, 6, 50, 51, 52, and 53). It cannot be assumed, however, that the materials requiring large thicknesses at stresses from 2 to 4 pounds per square inch will also require large

Table 1.--Results of a cushion design problem
applied to various materials¹

Material			Strain	Energy per	Cushion
				unit volume	thickness
Curled hair : Fiberglass : Shredded :					required
and latex : : rubber :					
			Percent	In.-lb./in. ³	In.
	5		84	0.18	10.4
1			80	.31	6.1
2			75	.46	4.1
3			69	.46	4.1
4			61	.45	4.2
		50	53	.37	5.1
		51	47	.32	5.9
		52	40	.37	5.1
		53	33	.34	5.5
	6		33	.33	5.7
	7		13	.13	14.4
	8		5	.06	31.3

¹The results summarized here apply to the problem given on page 12 where
W = 2.5 pounds, h = 30 inches, A = 40 square inches and G = 32.
The stress is therefore 2 pounds per square inch.

thickness at all ranges of stress. For example, in the range of stress from 20 to 40 pounds per square inch it can be shown by comparing the curves in figure 24 that materials 7, 8, and 9 would require less thickness than 50, 51, 52, and 53.

ADDITIONAL DESIGN TECHNIQUES

Several refinements to the method shown on page 12 for cushion thickness design can be accomplished by proper interpretation of the stress-strain curves. For the purpose of illustrating these techniques, consideration of resiliency will be omitted.

Energy

As has already been pointed out, a cushioned article continues to fall after the container strikes the floor. Therefore, the energy absorbed per unit volume of cushion (equation 18) is actually

$$e_m = \frac{W(h + S_m)}{AT_1} \quad (24)$$

where T_1 is the thickness of cushion required when the cushion displacement is included in the energy equation. If the height of drop, h , is much larger than the displacement, S , then the displacement can be neglected in equation (24). In general, this is true since the height of drop is taken as 30 to 36 inches, and the total cushion thickness is generally less than 5 inches. If $h < 10S$, then equation (24) should be used. Substituting $s_m T_1$ for S_m and solving for T_1 , we obtain

$$T_1 = \frac{Wh}{Ae_m - Ws_m} \quad (25)$$

Using equation (25), the thickness required in the example on page 12 becomes

$$T_1 = \frac{Wh}{Ae_m - Ws_m} = \frac{2.5 \text{ lb.} \times 30 \text{ in.}}{40 \text{ in.}^2 \times 0.31 \frac{\text{in.-lb.}}{\text{in.}^3} - 2.5 \text{ lb.} \times 0.8} = 7.2 \text{ in.}$$

Thus, for the illustration given, equation (25) should be used since an extra 1.1 inches or an increase of 18 percent in the cushion thickness is indicated.

Minimum Thickness

The thickness formula, equation (20), can be written in a different form by combining equations (20) and (21). Thus,

$$T = \frac{f_m h}{e_m G_m} \quad (26)$$

Since for a given cushion thickness design problem, the height of drop and the g-value are fixed, it can be seen from equation (26) that the thickness is directly proportional to the ratio, f/e . Figure 25 shows a plot of the ratio of stress to energy per unit volume versus stress for the stress-strain curve given in figure 4. The minimum point for this ratio is typical of many cushioning materials. Design at this f/e minimum to obtain minimum thickness was first suggested by R. R. Janssen(2).

In addition to the height of drop and the g-value, the weight of the article is also fixed for a given problem. Thus, the only remaining variable to change the point of operation on a given stress-strain curve is the

bearing area of the cushion. The cushion bearing area that will give minimum cushion thickness for a given article and cushion is obtained by solving equation (21) for A_2 , thus

$$A_2 = \frac{WG_m}{f_2} \quad (27)$$

Where subscript 2 indicates a value of a quantity corresponding to the minimum f/e ratio for a given cushion. Whether it is advantageous to use this minimum thickness will depend partly on the relation of the bearing area of the cushion to the area of the article.

Let f_a represent the stress for a given article for which the area of the article is equal to the bearing area of the cushion. If f_a is less than f_2 , then by decreasing the area of the cushion, f_a can be increased to f_2 . The result can be a considerable decrease in cushion thickness since the f/e ratio drops very rapidly at stresses less than f_2 . Since the area of the cushion is less than the area of the article, the decrease in cushion thickness will result in decreased container volume. Certain practical limitations exist for this condition. As the cushion area is reduced, the problem of stability of columns of cushions becomes increasingly difficult to handle. For some articles, the g -value could be dependent on the bearing surface.

If f_a is greater than f_2 it is necessary to make the bearing area of the cushion larger than the area of the article to decrease the stress to f_2 . Thus, the decrease in cushion thickness will have to decrease the dimensions of the pack more than the increase in bearing area tends to increase the dimensions of the pack if there is to be any saving in cube. A more complete discussion of this technique is contained in the reports by Lt. Roger Orensteen(7).

Precompression and Compression

A cushion is called precompressed when it is held at a fixed strain by a clamping or quilting device. A cushion is called compressed when it is given an initial strain during packing because the cushion is thicker than the space provided between article and container. Design under these conditions can be based on the stress-strain curve obtained for an uncompressed cushion provided compression or precompression over a period of time do not appreciably change the characteristics of the stress-strain curve.

By precompressing a cushion, the low energy absorption portion of the stress-strain curve is eliminated. Let subscript p indicate values of quantities at the strain to which a given cushion has been precompressed. Then the total energy absorbed per unit volume of cushion, e_t , will be

$$e_t = e_m - e_p \quad (28)$$

and the total thickness of the cushion will be

$$T = \frac{Wh}{Ae_t} \quad (29)$$

The precompressed cushion thickness, T_3 , will be

$$T_3 = T - s_p T \quad \text{or} \quad T_3 = \frac{Wh}{Ae_t} (1 - s_p) \quad (30)$$

As an illustration, precompress the cushion used as an example on page 12 to a strain of 0.3 (see fig. 4).

Then

$$T_3 = \left(\frac{2.5 \text{ lb.} \times 30 \text{ in.}}{40 \text{ in.}^2 \times 0.28 \frac{\text{in.-lb.}}{\text{in.}^3}} \right) (1 - 0.3) = 4.7 \text{ in.}$$

This decrease in thickness will not increase indefinitely since as $(1 - s_p)$ decreases, tending to decrease thickness, e_t decreases tending to increase thickness. This point is illustrated in the following tabulation:

s_p	T	T_3	Thickness efficiency
	In.	In.	Percent
0.0	6.1	6.1	15
.3	6.7	4.7	20
.5	8.1	4.1	23
.7	15.6	4.7	20

With compressed cushions, a stress is exerted on both the top and bottom of the article. Therefore, the stress-strain curves for both cushions must be added to obtain the actual stress acting on the article for each strain. Such an addition is illustrated in figure 26 for material No. 1 compressed to a strain of 0.30. Note that the range of stress for which the curve is essentially linear is increased. Also the slope of the linear portion of the curve is increased. This result is pointed out by Marvin Masel(5).

Let subscript c indicate values of quantities at the strain to which a given cushion has been compressed. Then the total energy absorbed per unit volume of cushion e_t , will be

$$e_t = e_m - 2e_c \quad (31)$$

$$\text{where } s_m > 2s_c$$

For $s_m < 2s_c$

$$e_t = e_m - 2e_c + e_{2c} - m \quad (32)$$

The total thickness of the cushion will be

$$T = \frac{Wh}{Ae_t} \quad (33)$$

The compressed thickness of the cushion, T_4 , will be

$$T_4 = T - s_c T \quad \text{or} \quad T_4 = \frac{Wh}{Ae_t} (1 - s_c) \quad (34)$$

As an illustration, compress the cushion used as an example on page 12 to a strain of 0.30 (see figs. 4 and 26). Since

$$s_m = 0.8 \text{ and } s_c = 0.30, s_m > 2s_c, \text{ and}$$

$$e = 0.31 \frac{\text{in.-lb.}}{\text{in.}^3} - 2 \times 0.02 \frac{\text{in.-lb.}}{\text{in.}^3} = 0.27 \frac{\text{in.-lb.}}{\text{in.}^3}$$

The thickness of the compressed cushion will be

$$T_4 = \left(\frac{2.5 \text{ lb.} \times 30 \text{ in.}}{40 \text{ in.}^2 \times 0.27 \frac{\text{in.-lb.}}{\text{in.}^3}} \right) (1 - 0.30) = 4.9 \text{ in.}$$

Again a reduction in thickness required is achieved. For the same reason as discussed and illustrated for precompressed cushions, additional compression will not continue to increase cushion thickness efficiency.

Cushioning materials are often compressed during packing to prevent looseness in a pack after several drops. It is thought, however, that looseness of itself can cause no damage provided the article cannot reorient itself in such a manner as to change the bearing area. Although the energy

absorbing capacity of many cushions is changed as a given cushion changes thickness due to repeated displacements, compression cannot improve this characteristic.

Graduated Density

It is often suggested that a cushion made up of layers of various densities of a given type cushion would provide, in general, increased protection to a given article. For an article with a flat bearing surface and a fragility given by a g-factor, it can be shown that this would result in thicker cushions.

Stress-strain curves for two materials must be combined at equal stresses rather than equal strains. Let subscript 1 indicate quantities referring to material No. 1, subscript 4 indicate quantities referring to material No. 4 and subscript t indicate quantities referring to a cushion made up of a layer of material No. 1 and a layer of material No. 4. At a given stress the strain will be

$$s_t = \frac{S_1 + S_4}{T_1 + T_4} \quad (35)$$

$$\text{But } S_1 = s_1 T_1 \quad \text{and} \quad S_4 = s_4 T_4 \quad (36)$$

so from equations 35 and 36 we obtain,

$$s_t = \frac{s_1 T_1 + s_4 T_4}{T_1 + T_4} \quad (37)$$

The energy absorbed by the combined cushions, E_t , at a given stress will be

$$E_t = E_1 + E_4 \quad (38)$$

$$\text{but } e_1 = \frac{E_1}{AT_1} \quad \text{and} \quad e_4 = \frac{E_4}{AT_4} \quad (39)$$

so combining equations 38 and 39 yields

$$E_t = A (e_1 T_1 + e_4 T_4) \quad (40)$$

$$\text{Since} \quad e_t = \frac{E_t}{A (T_1 + T_4)} \quad (41)$$

substituting equation 40 in 41 gives

$$e_t = \frac{e_1 T_1 + e_4 T_4}{T_1 + T_4} \quad (42)$$

In figure 27 is shown the stress-strain curves for materials No. 1, No. 4 and a cushion made up of equal thicknesses of materials 1 and 4.

When $T_1 = T_4$ equation 37 becomes

$$s_t = \frac{s_1 + s_4}{2} \quad (43)$$

and equation 42 becomes

$$e_t = \frac{e_1 + e_4}{2} \quad (44)$$

From equation 44 it can be seen that e_t will always be less than the larger of e_1 and e_4 except when $e_1 = e_4$ at which point $e_t = e_1 = e_4$. This is shown graphically in figure 28 where stress is plotted against energy per unit volume. Figure 28 shows that at a given stress a cushion made up of two different densities will always require a thickness greater than or equal to the thickness required for one of the densities of cushion from which it was made.

A graduated density cushion can be used advantageously to distribute the stress on an irregular object. This can also be accomplished through molding or die cutting of cushioning materials.

NOTATION

A = area in square inches
a = acceleration in feet per second per second
E = energy in inch pounds
e = energy per unit volume in inch pounds per cubic inch
F = force in pounds
f = stress in pounds per square inch
G = g-value, dimensionless
g = acceleration due to gravity, 32.2 feet per second per second
h = height of drop in inches
K' = spring constant in pounds for a cushion with ideal elasticity
K'' = spring constant in pounds per inch for a cushion with linear elasticity
m = mass in slugs
S = displacement in inches
s = strain in inch per inch
T = measured thickness of a cushion in inches
T' = thickness in inches of a cushion with ideal elasticity
T'' = thickness in inches of a cushion with linear elasticity
W = weight in pounds

Subscripts -

m = maximum value for a given problem. Other subscripts are defined in the particular section in which they are used.

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The approach to cushion thickness design taken by Mr. Gretz is basically the same as that given in this report. Although the calculations would appear to be quite different, they are equivalent.

2. Janssen, R. R. Selecting Package Cushioning Materials to Provide Minimum Container Cubage. North American Aviation Corp., Los Angeles, Calif. October 1952.

In this paper Mr. Janssen shows that there is a point on curves typical of many cushioning materials at which design will yield minimum cushion thickness. Consideration of the circumstances under which design at this point is advantageous has been given by the authors of several reports referenced here.

3. Kellicutt, K. Q. Application of the Properties of Cushioning Materials in the Design of Cushions. Forest Products Laboratory Report No. R1627. September 1946.

This study, conducted in cooperation with the Air Force, was a continuation of the work reported in Forest Products Laboratory Report No. R1489. While additional tests and data are presented, the method of calculating cushion thickness is the same.

4. Lassen, L.; Kellicutt, K. Q.; and Sanderson, W. J. Development of a Procedure for the Determination of the Properties of Cushioning Materials and Their Application in the Design of Cushions. Forest Products Laboratory Report No. R1489. June 1945.

It is believed that this study, conducted in cooperation with the Air Force, was one of the initial comprehensive investigations on the subject of package cushioning materials. Cushion thickness design is effected through the use of a family of approximately 10 energy curves for each different cushioning material.

5. Masel, M. A Mathematical Approach to Cushioning. Modern Packaging. May 1953.

The approach to package cushion design taken by Mr. Masel is basically the same as that of this report.

6. Mindlin, R. D. Dynamics of Package Cushioning. Bell Telephone Laboratories Technical Publications. Monograph B-1369. July 1945.

This paper by Mr. Mindlin represents the most complete mathematical treatment of package cushioning at this time. Part I of this paper is concerned primarily with the methods of fitting experimentally obtained curves for cushioning materials to five analytical equations for the purpose of predicting the maximum acceleration and displacement of a given packaged article for a specific cushion. This procedure has the advantage of permitting direct mathematical analysis in Parts II through IV of this paper. The numerical method shown by Mr. Mindlin for cushions having anomalous elasticity represents the type of approach being taken in this report. The disadvantages pointed out for this method by Mr. Mindlin have been overcome.

7. Orensteen, R. B. The Selection of Cushion Area in the Design of Package Cushioning. WADC Technical Report 53-43, U. S. Air Force, Wright Air Development Center, March 1953, and A Technique for the Design of Glass Fiber Package Cushioning. WADC Technical Report 53-68, U. S. Air Force, Wright Air Development Center, March 1953.

The first of these reports shows the results of combining certain principles shown in this work and the minimum thickness suggested by Mr. Janssen. The other report demonstrates the application of these methods to fiberglass cushioning materials. Economical design for the factors of cushion thickness, density and area, are obtained graphically for a series of densities for fiberglass cushioning material.

8. Underhill, A. M. Basic Principles of Package Cushioning.
General Electric Private Publication. General Electric Company,
Schnectady, N. Y.

Mr. Underhill shows a method of estimating the type and thickness of cushioning material required based on a linear spring. The cushion design obtained in this way is then checked more accurately by using the methods of R. D. Mindlin for a cushioning material with cubic type elasticity.

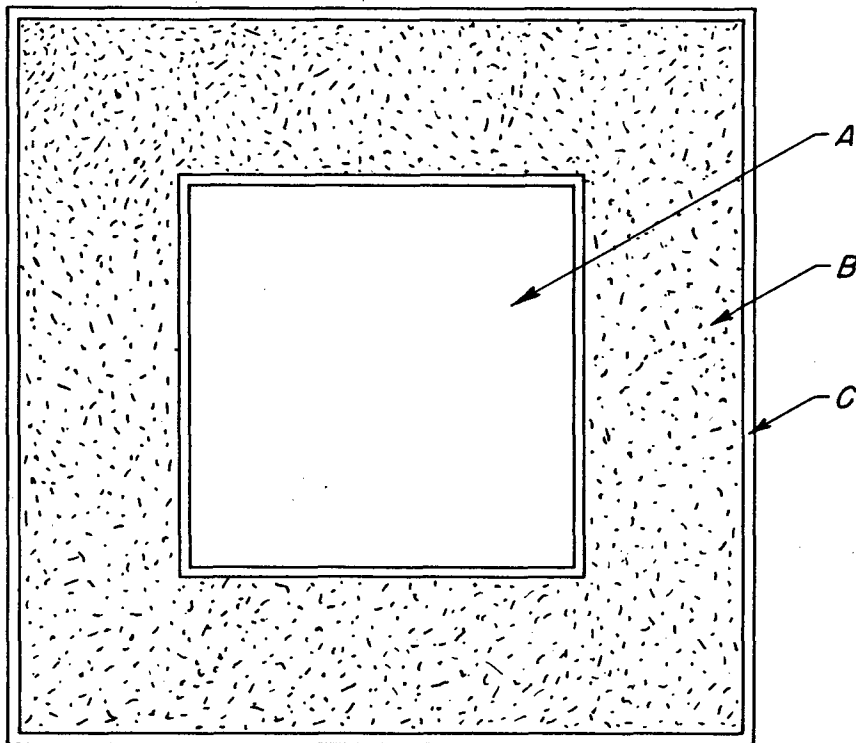


Figure 1.--A cushioned pack consists essentially of (A) an article, often in a chip-board or fiberboard container; (B) a cushioning material; and, (C) an outer container.

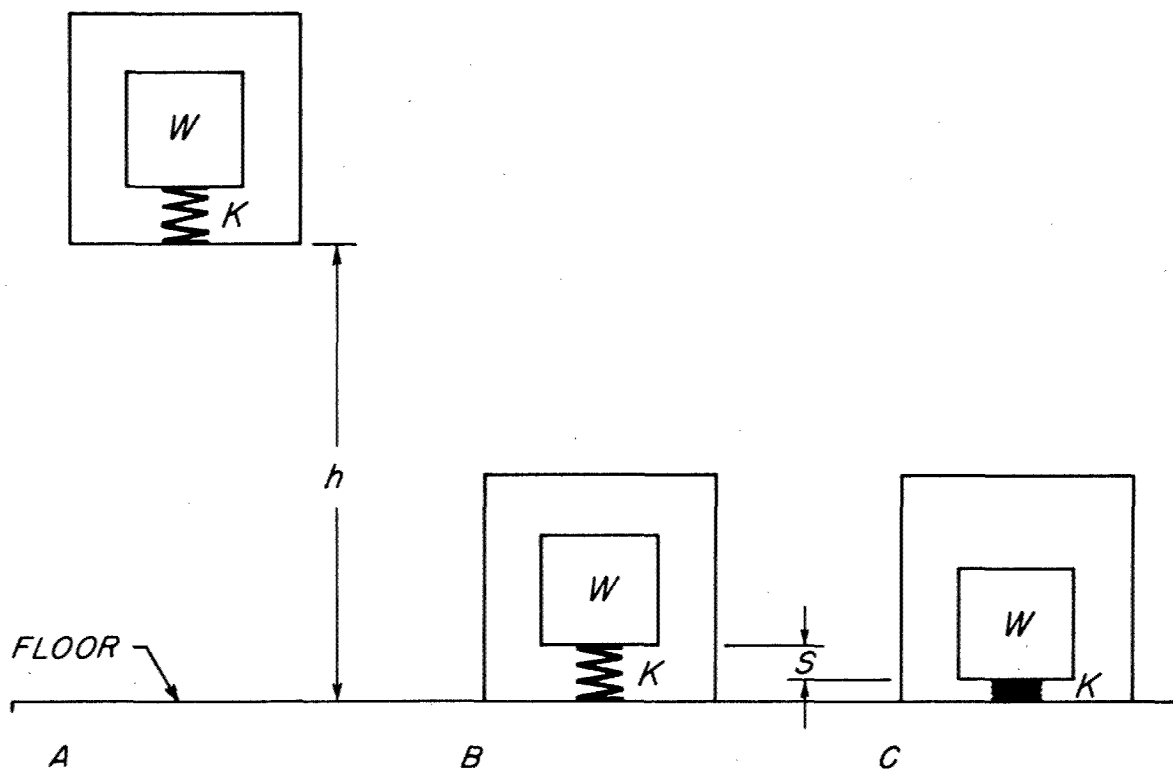


Figure 2.--An idealized representation of a cushioned pack during a drop.

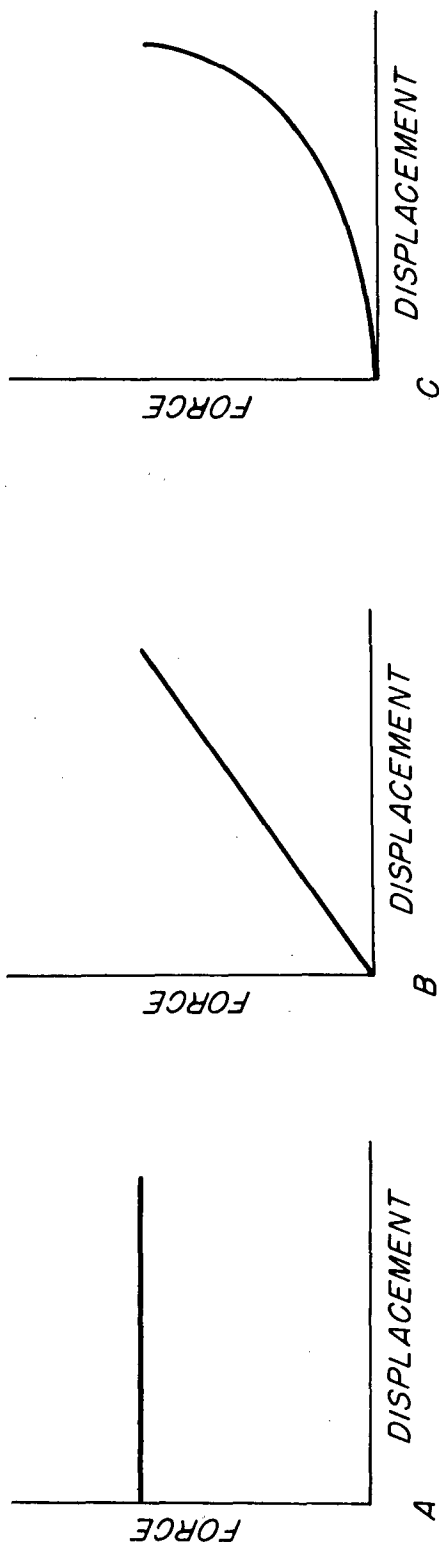


Figure 3.---Force-displacement curves of three types of cushioning materials. A, ideal elasticity; B, linear elasticity; and, C, anomalous elasticity.

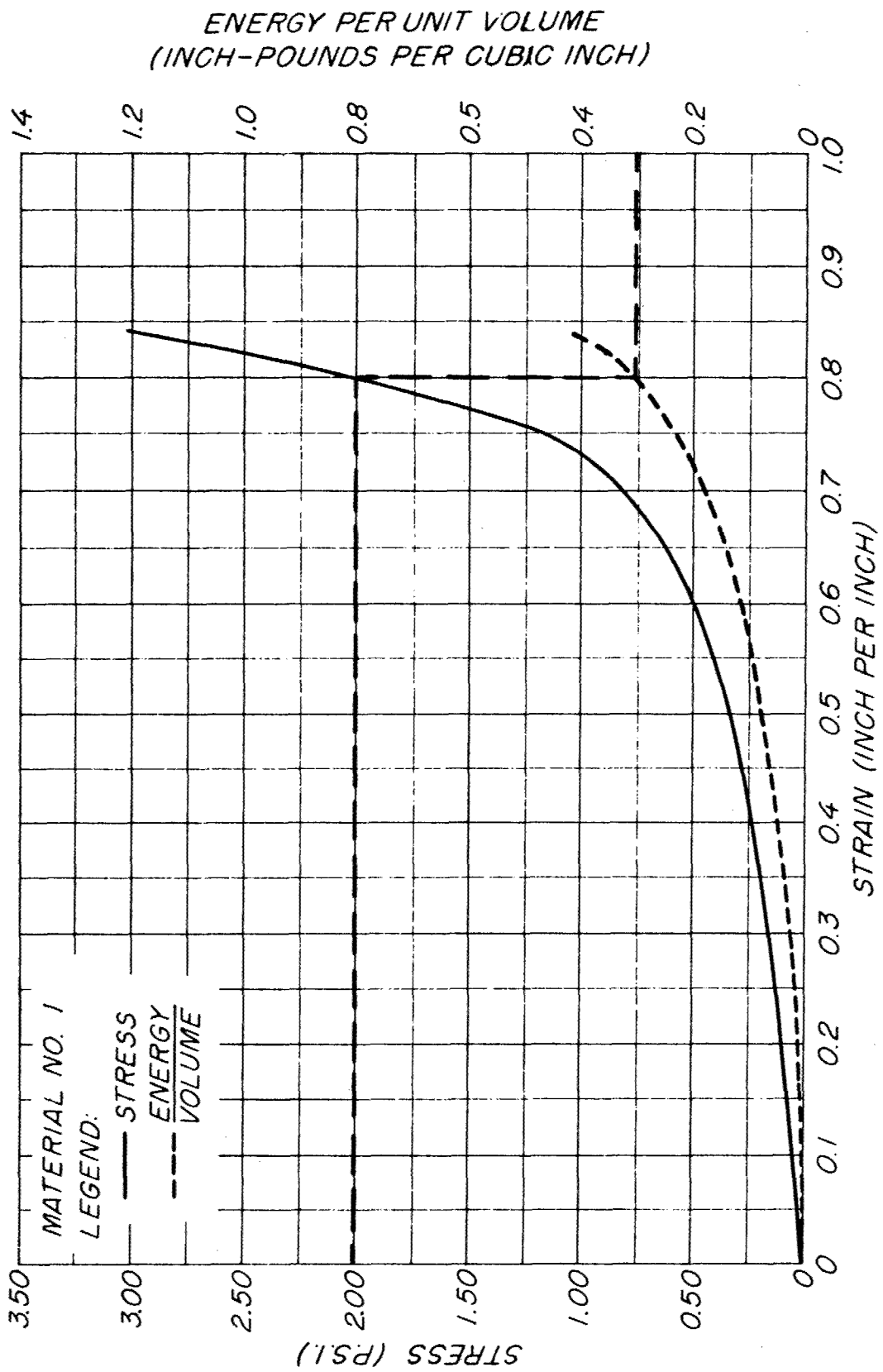
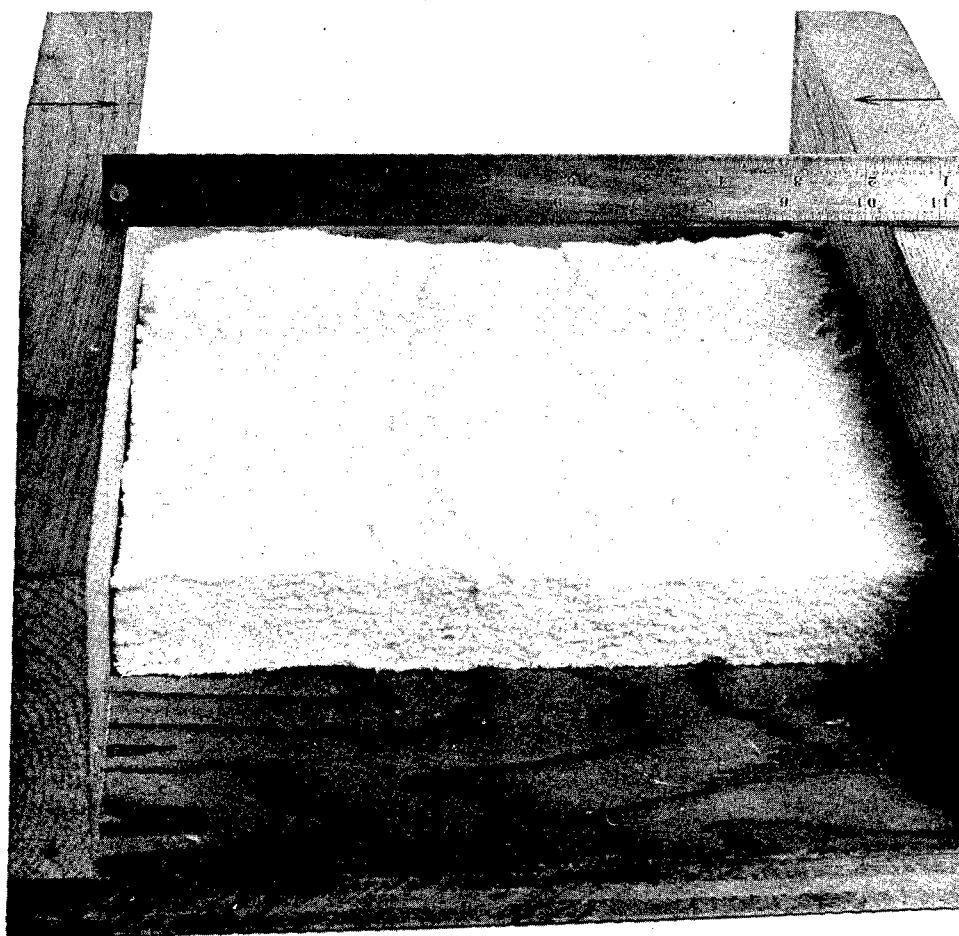
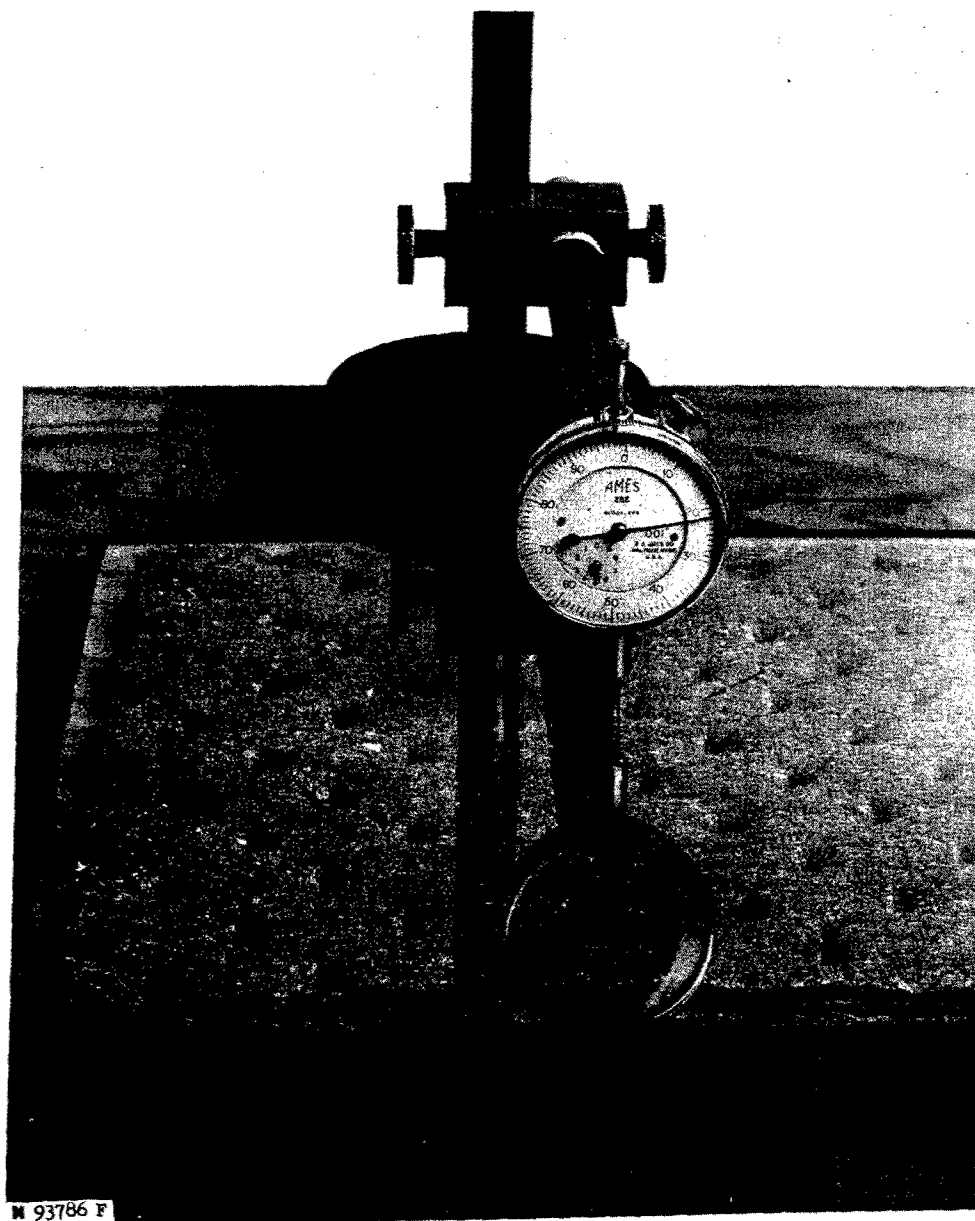


Figure 4.--Typical stress-strain and energy per unit volume-strain curves for anomalous elasticity.



M 93787 F

Figure 5.--Measuring jig used to determine
the length and width of a specimen.



M 93786 F

Figure 6.--This jig was used to measure the thickness of a specimen.



Figure 7.--Force-displacement test of cushion materials
were made on a universal testing machine.

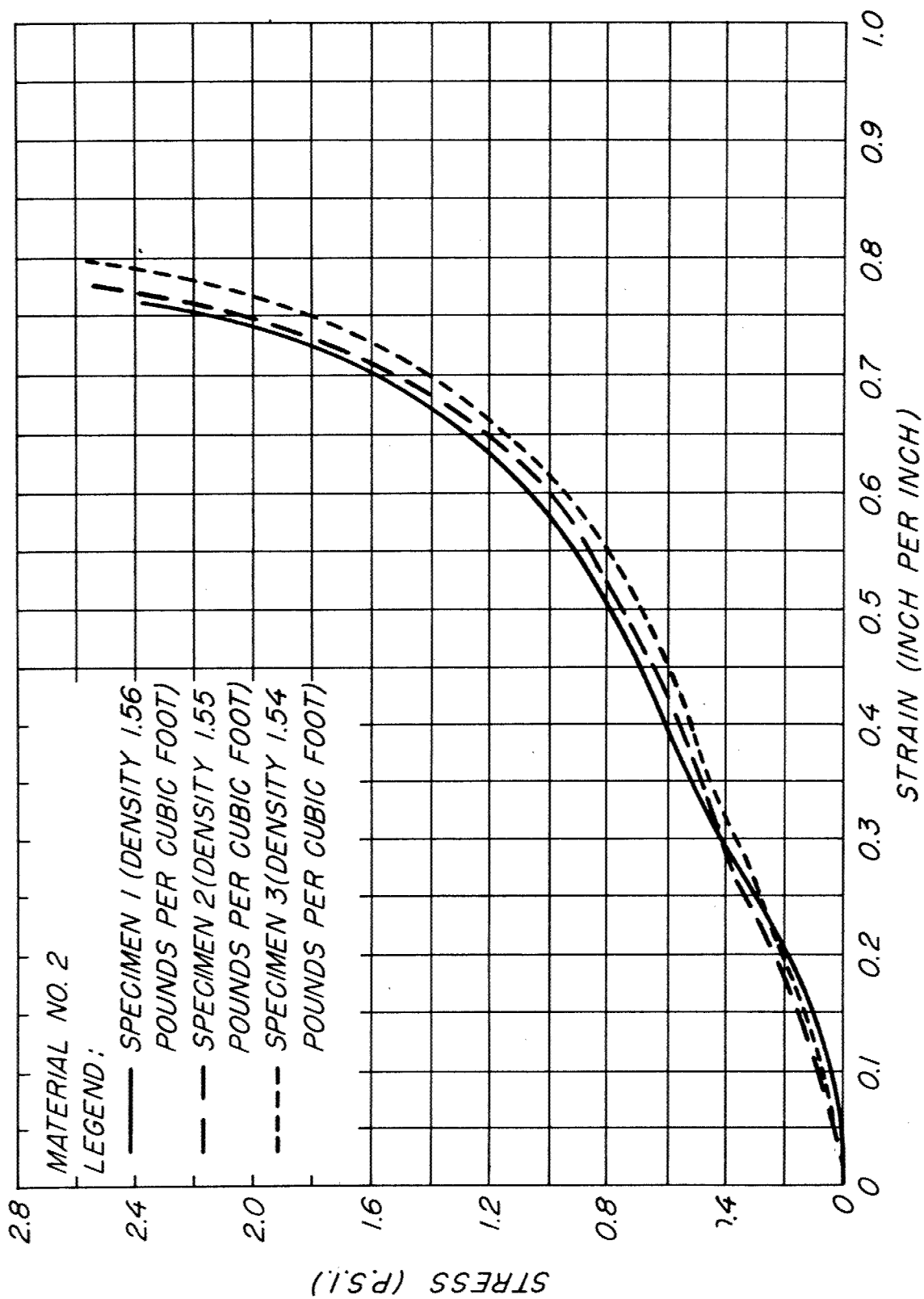


Figure 8.--Stress-strain curves of three specimens of material No. 2.

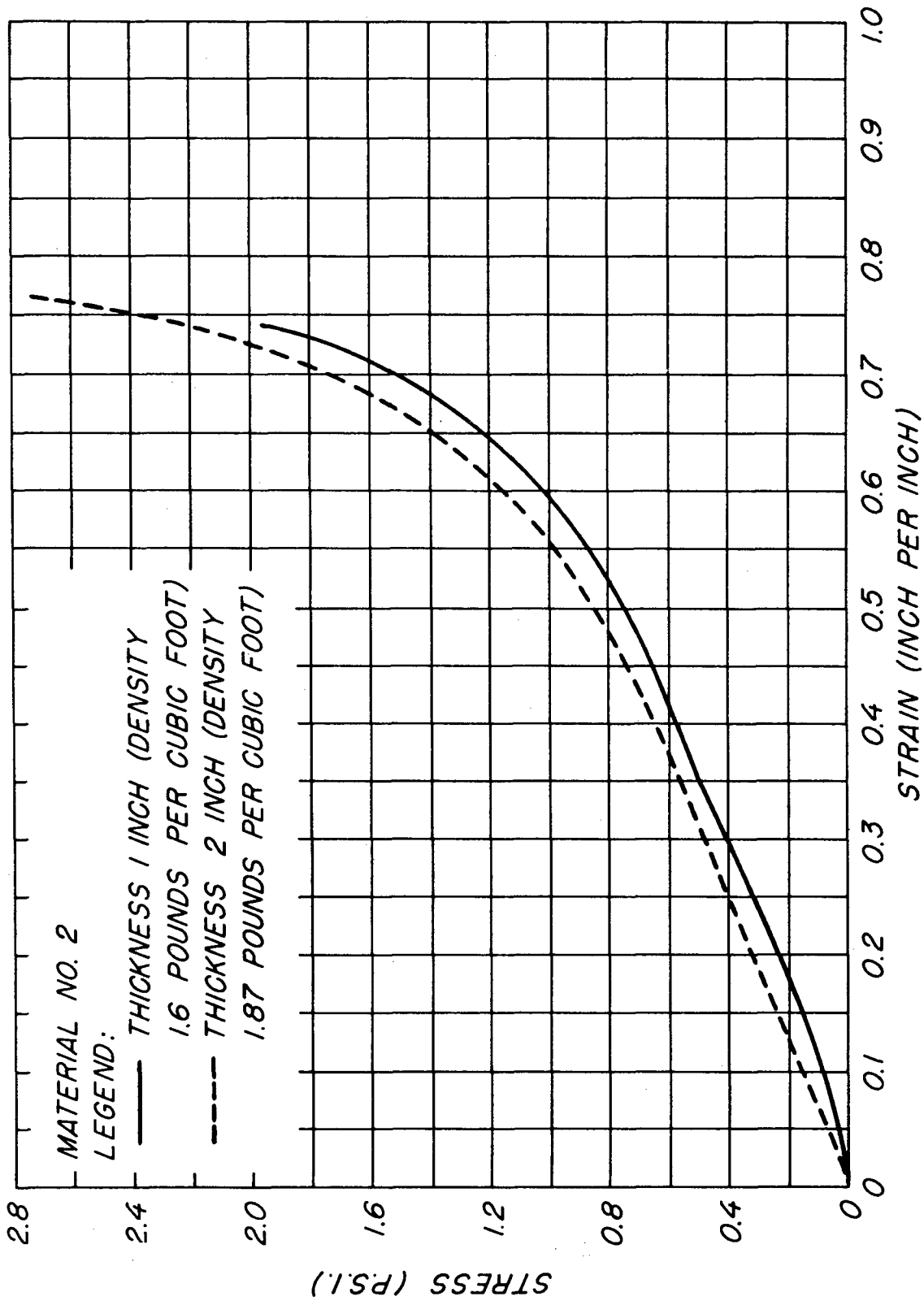


Figure 9.---Average stress-strain curves of two specimens of the same material with the same nominal density but of different thicknesses.

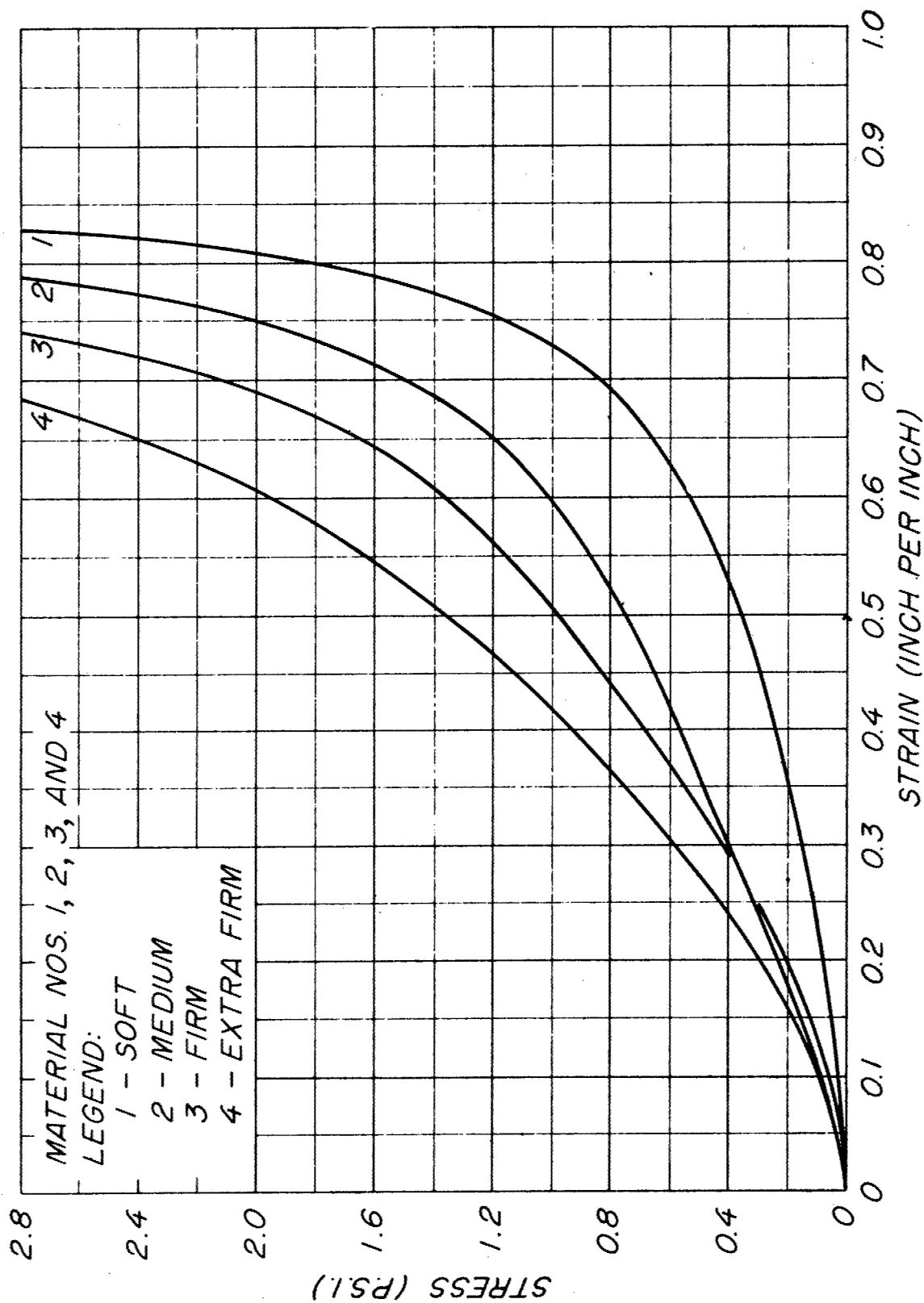


Figure 10.--For curled cattle hair, the stress increases at a given value of strain as the density of the material increases.

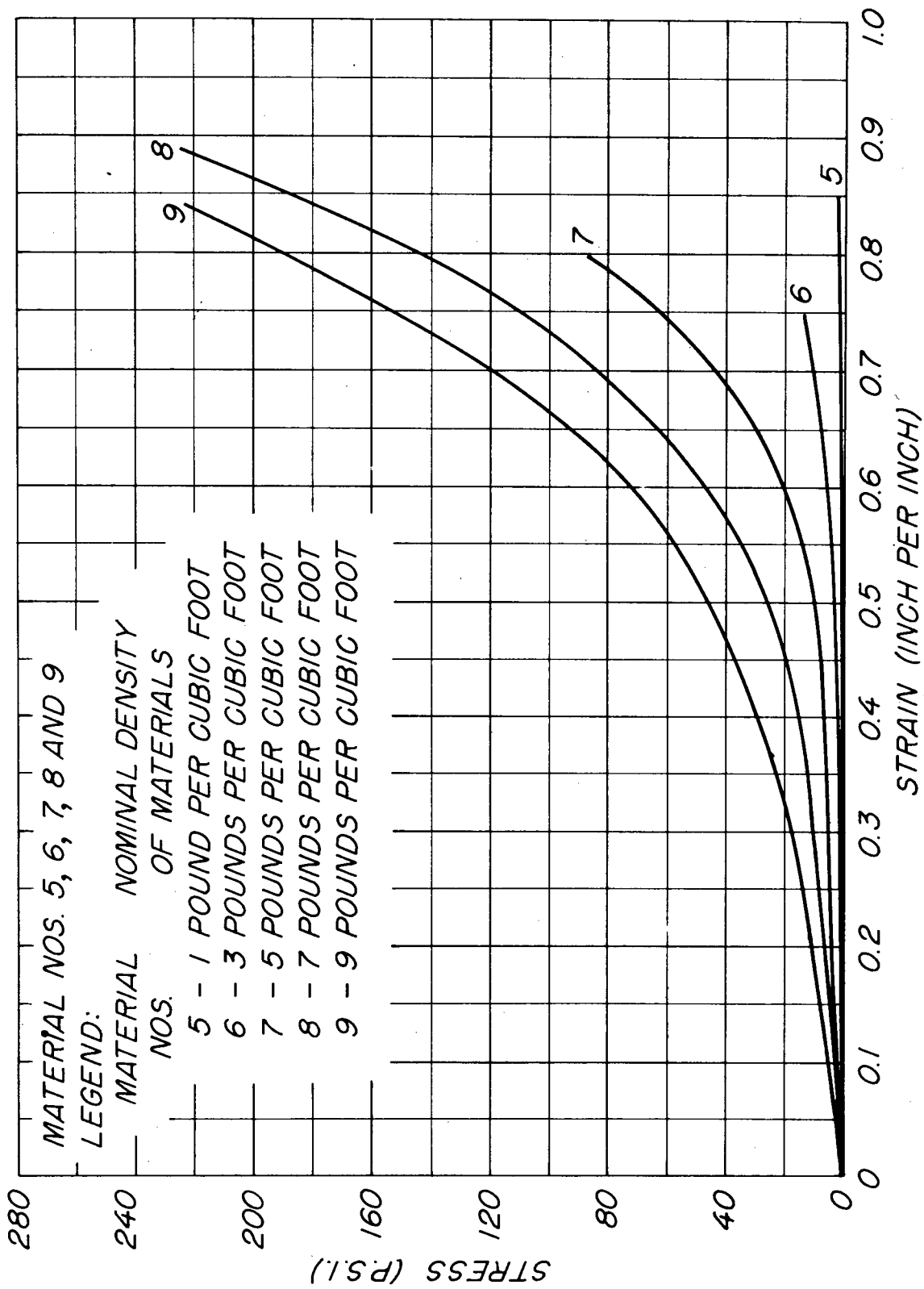


Figure 11.--For fiberglass materials the stress increases at a given value of strain as the density of the material increases.

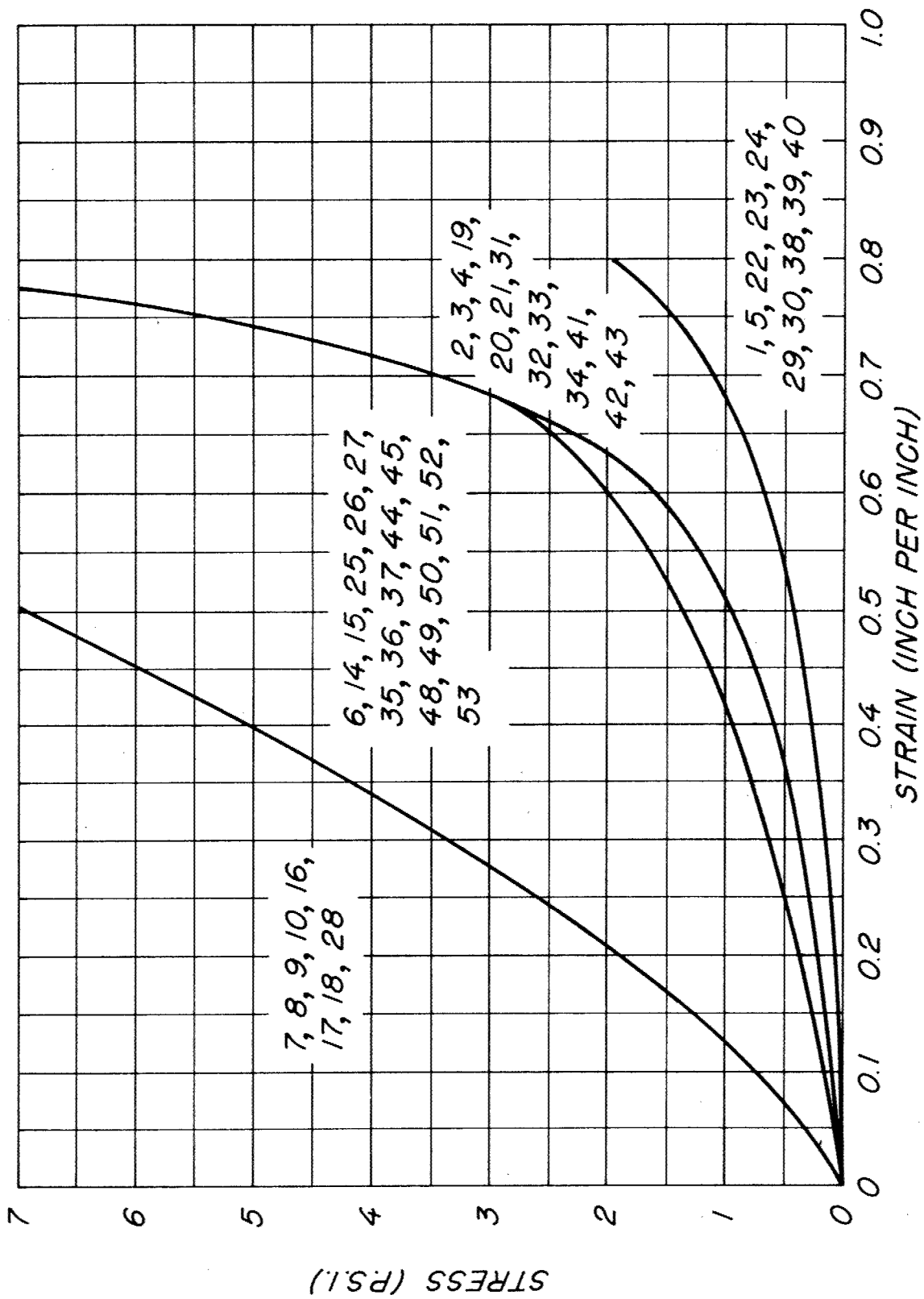


Figure 12.--Boundaries for stress-strain curves shown in figures 13, 14, 15, and 16.

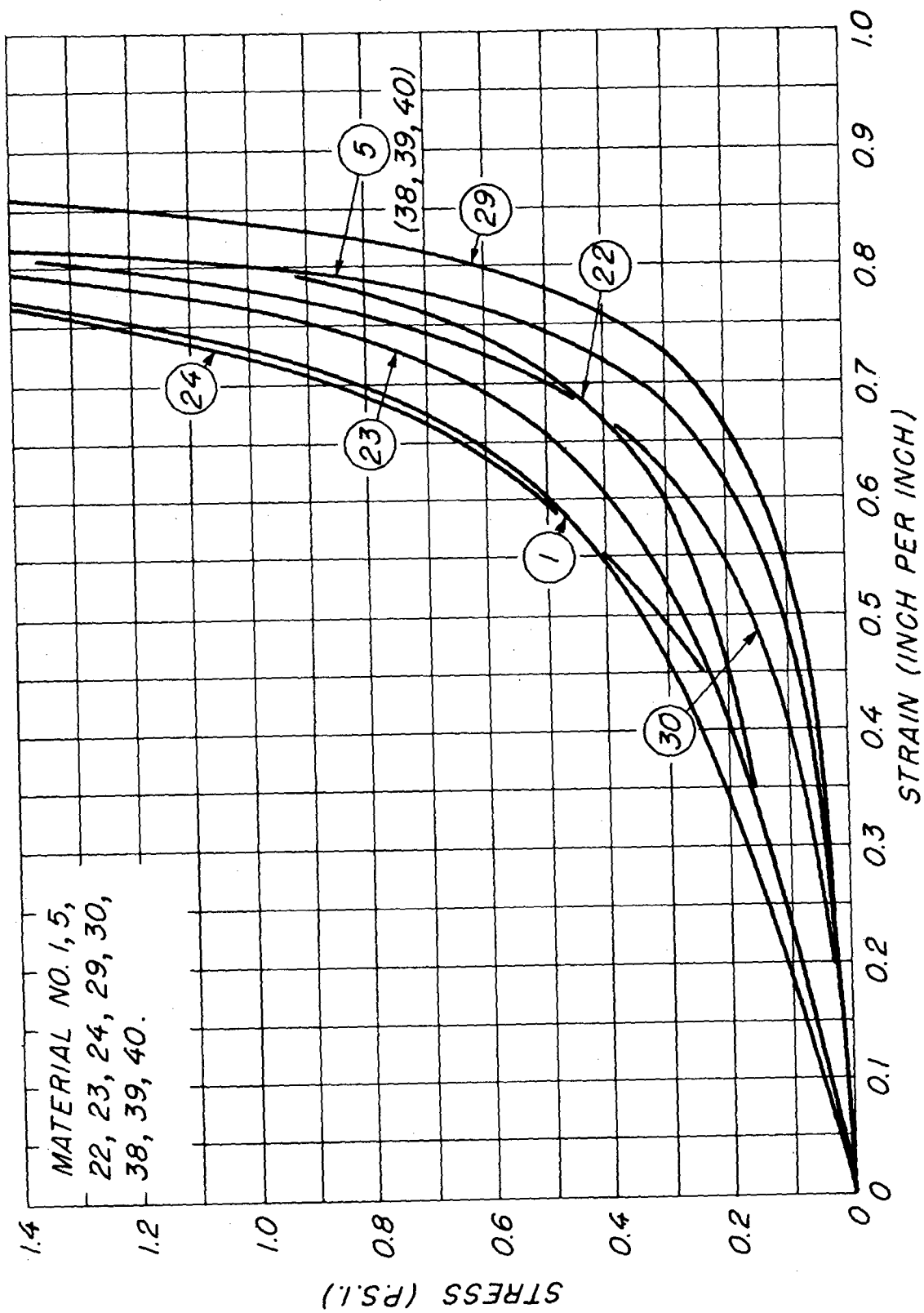


Figure 13.--Stress-strain curves for comparison of materials.

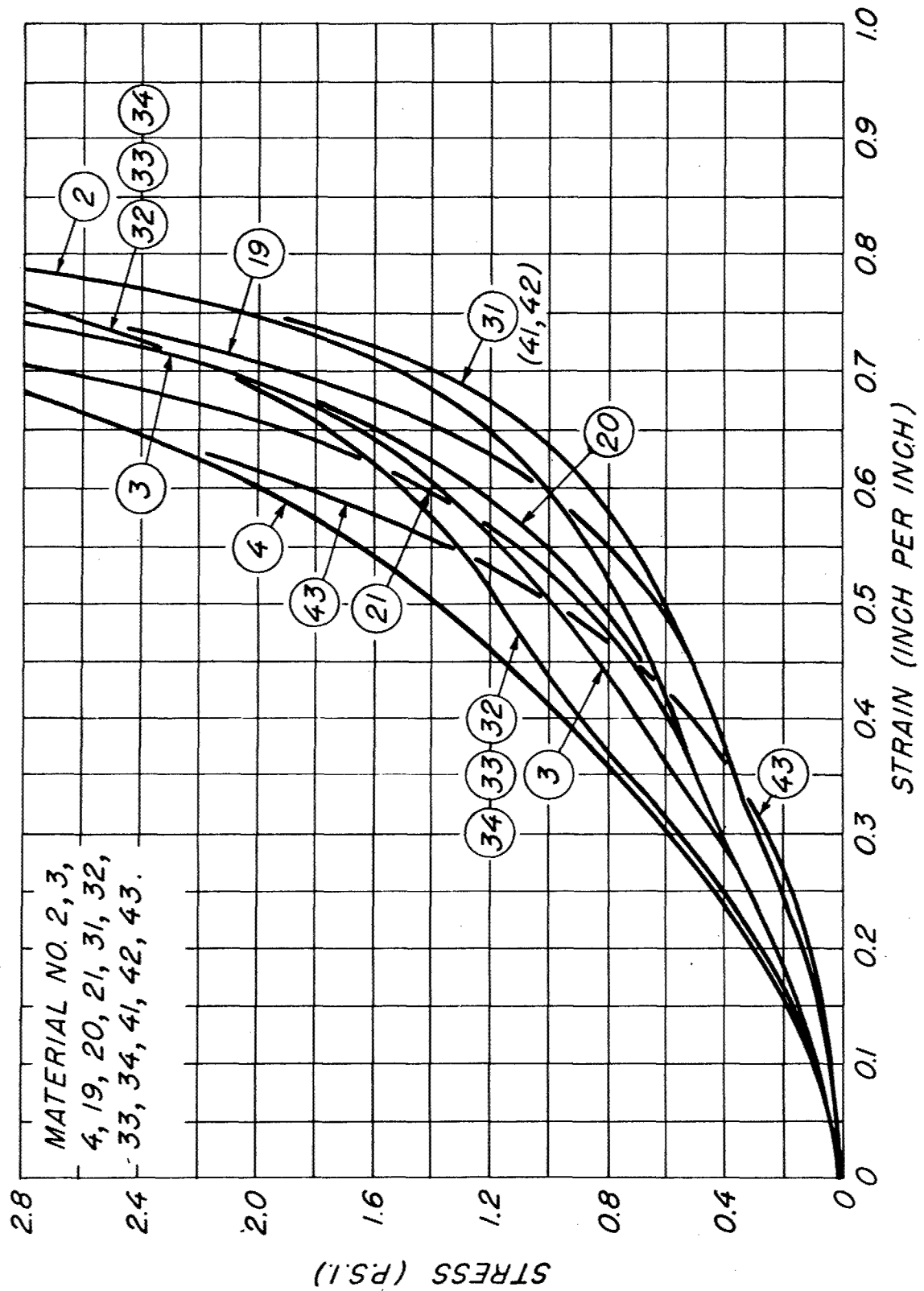


Figure 14.--Stress-strain curves for comparison of materials.

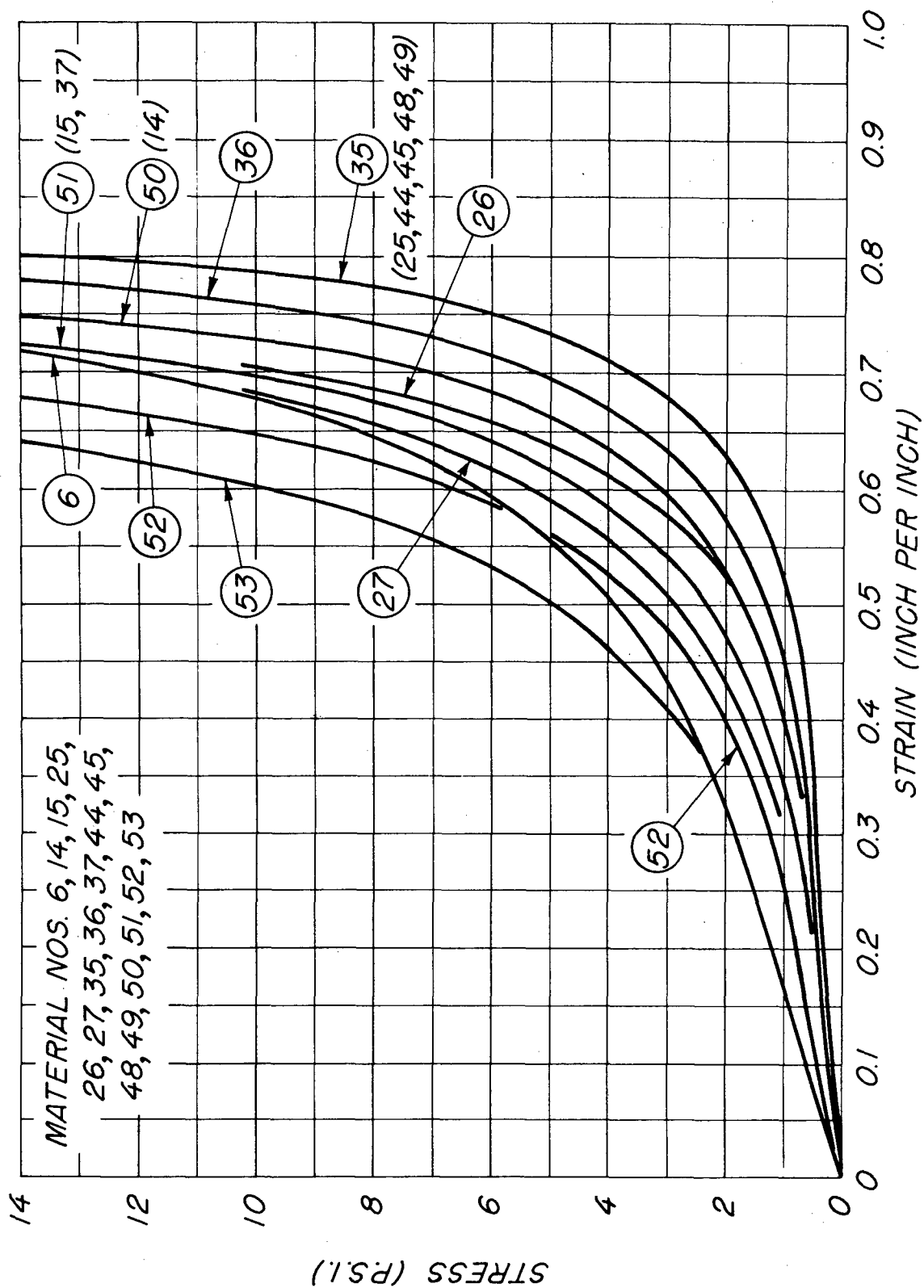


Figure 15.--Stress-strain curves for comparison of materials.

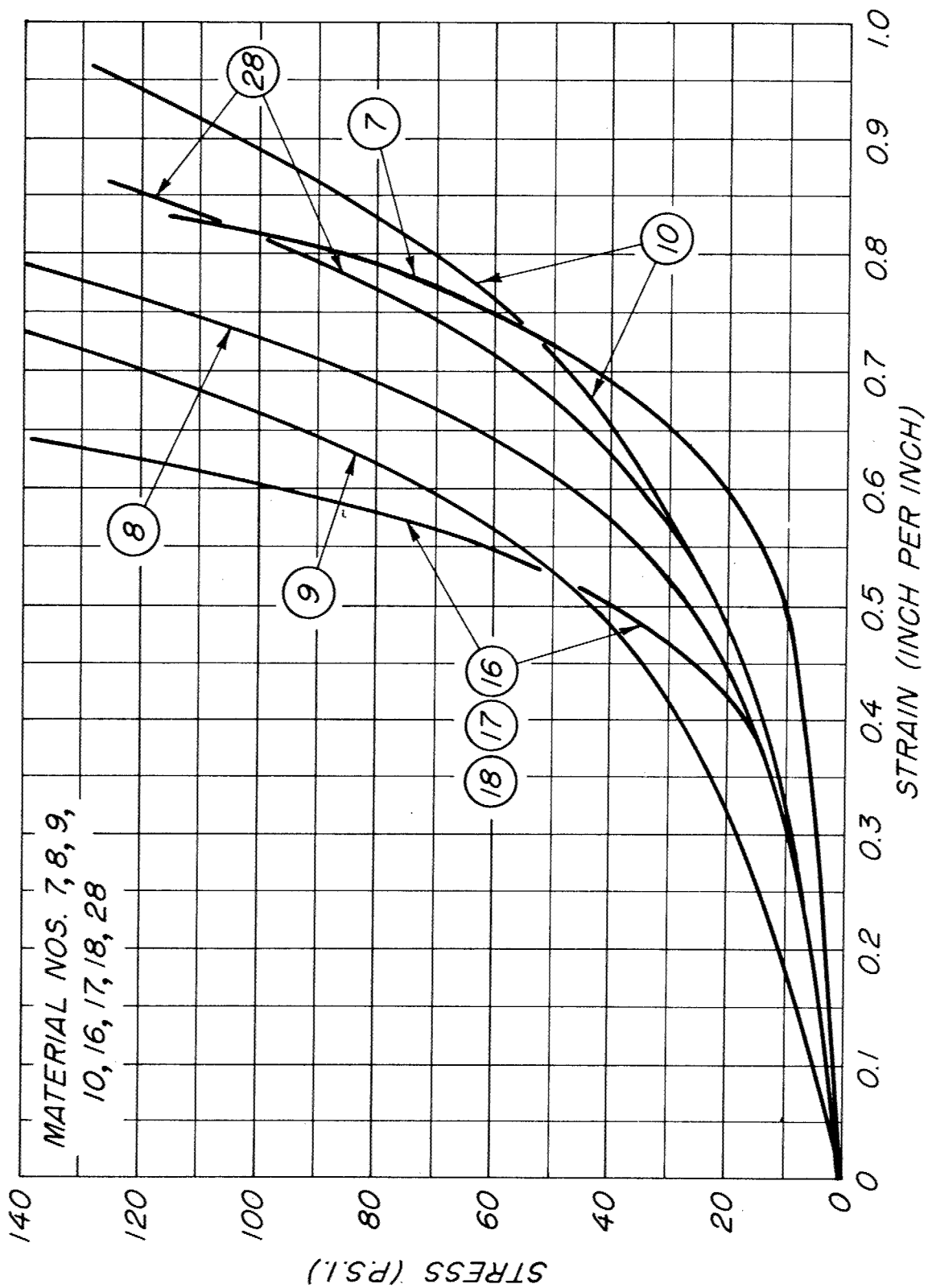
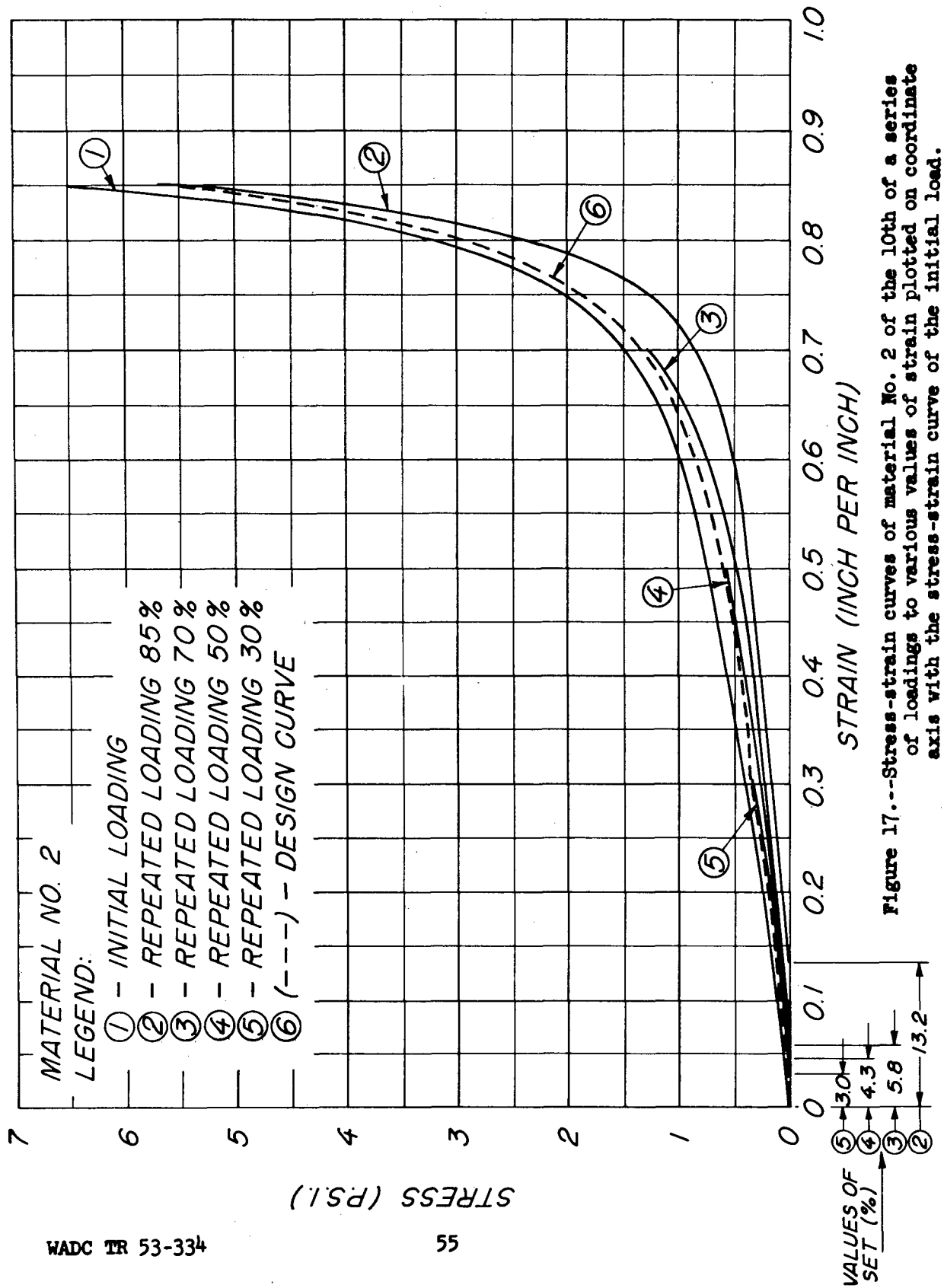
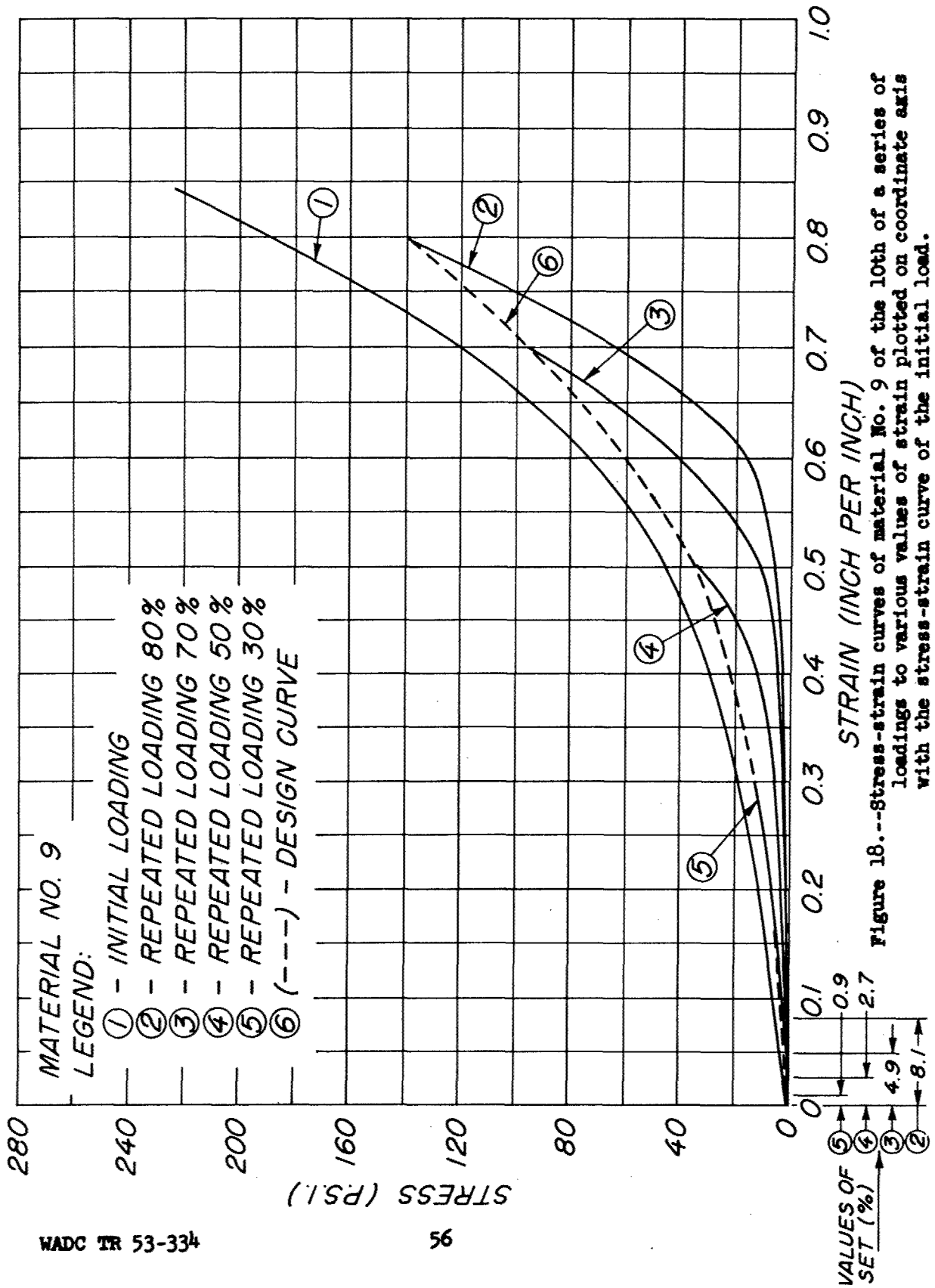


Figure 16.--Stress-strain curves for comparison of materials.





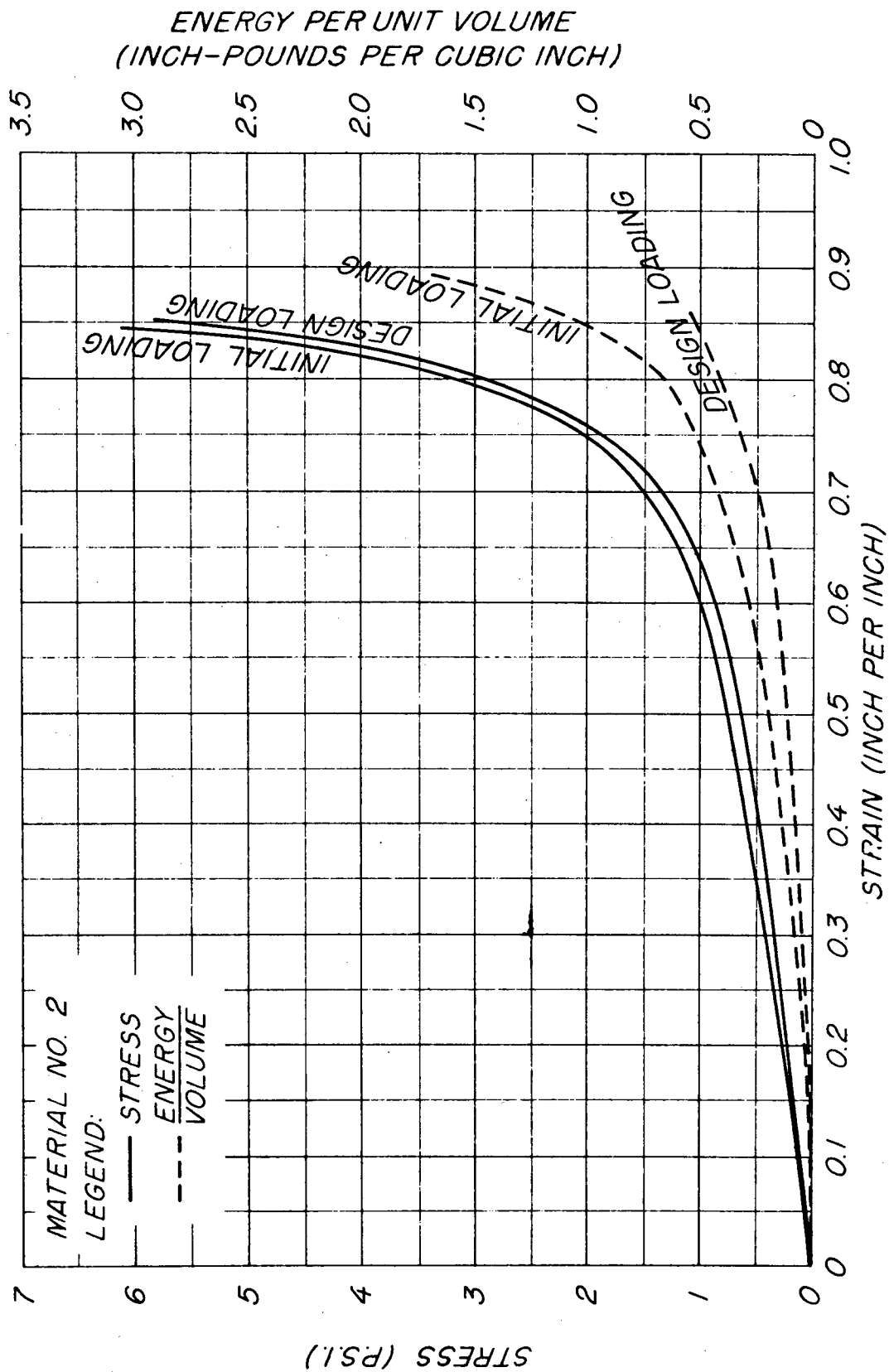


Figure 19.--Composite stress-strain and energy per unit volume-strain curves for initial and design loading of material No. 2.

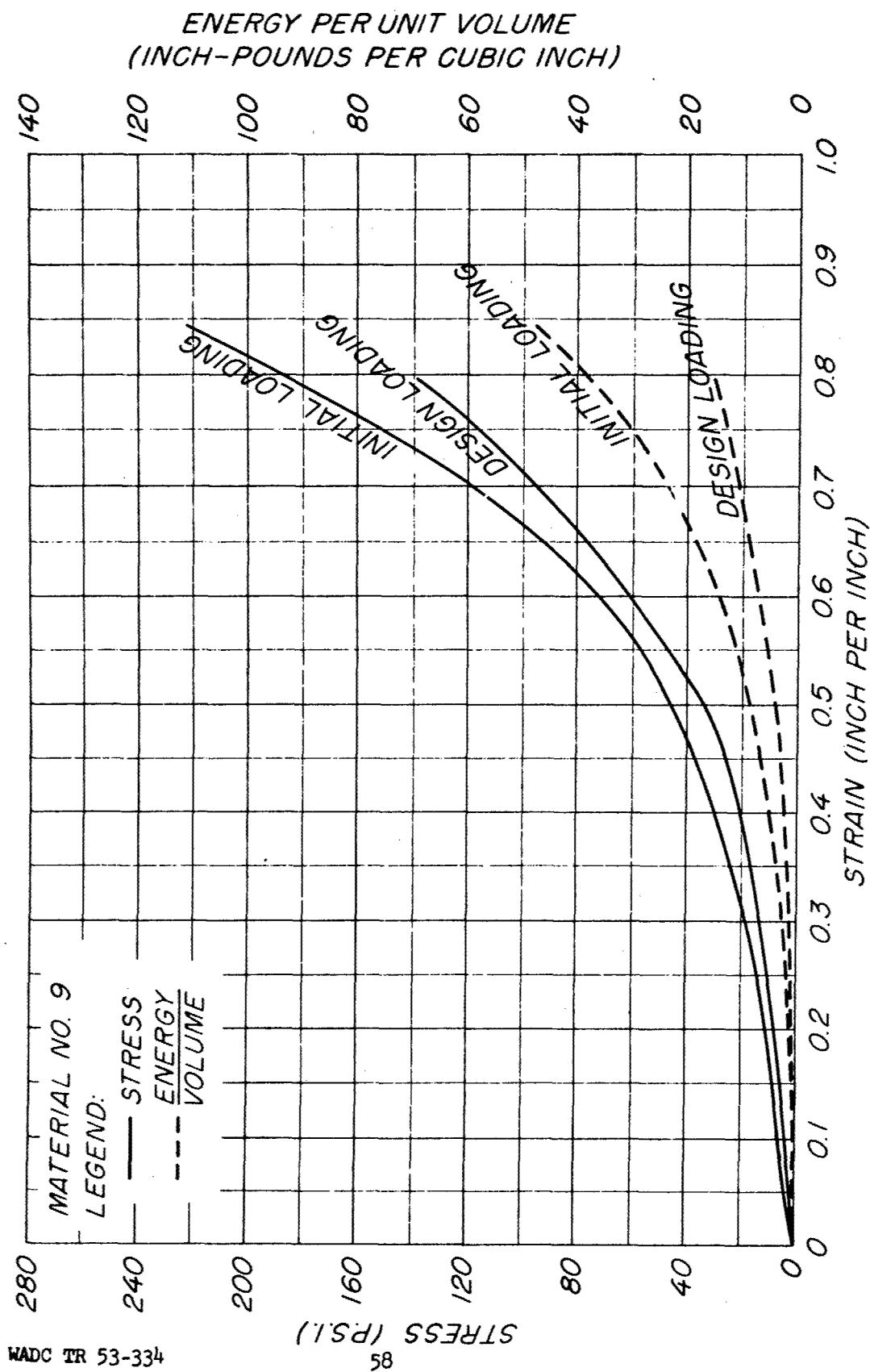


Figure 20.---Composite stress-strain and energy per unit volume-strain curves for initial and design loading of material No. 9.

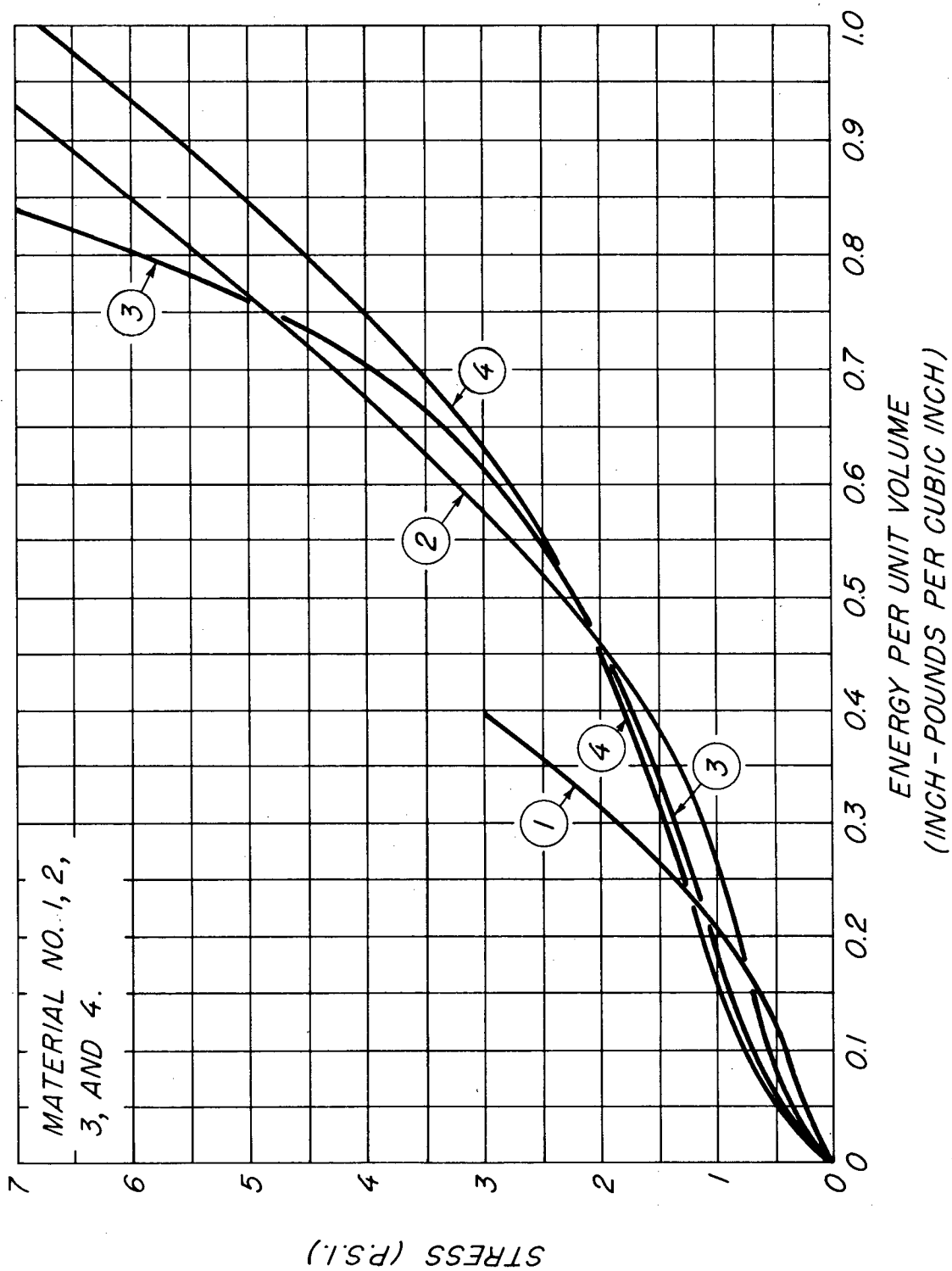


Figure 21.--Stress-energy per unit volume curves for materials 1, 2, 3, and 4.

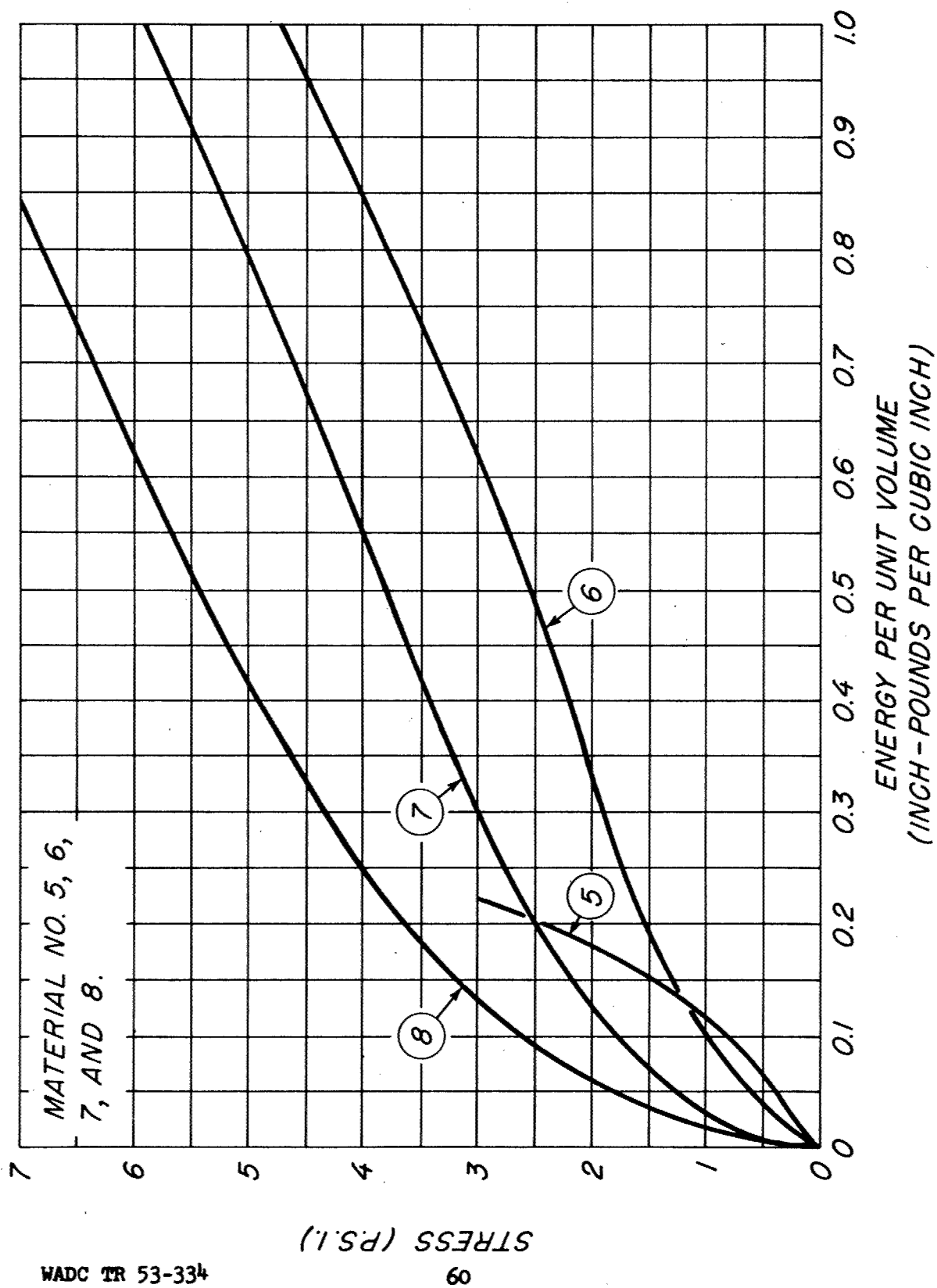


Figure 22.--Stress-energy per unit volume curves for materials 5, 6, 7, and 8.

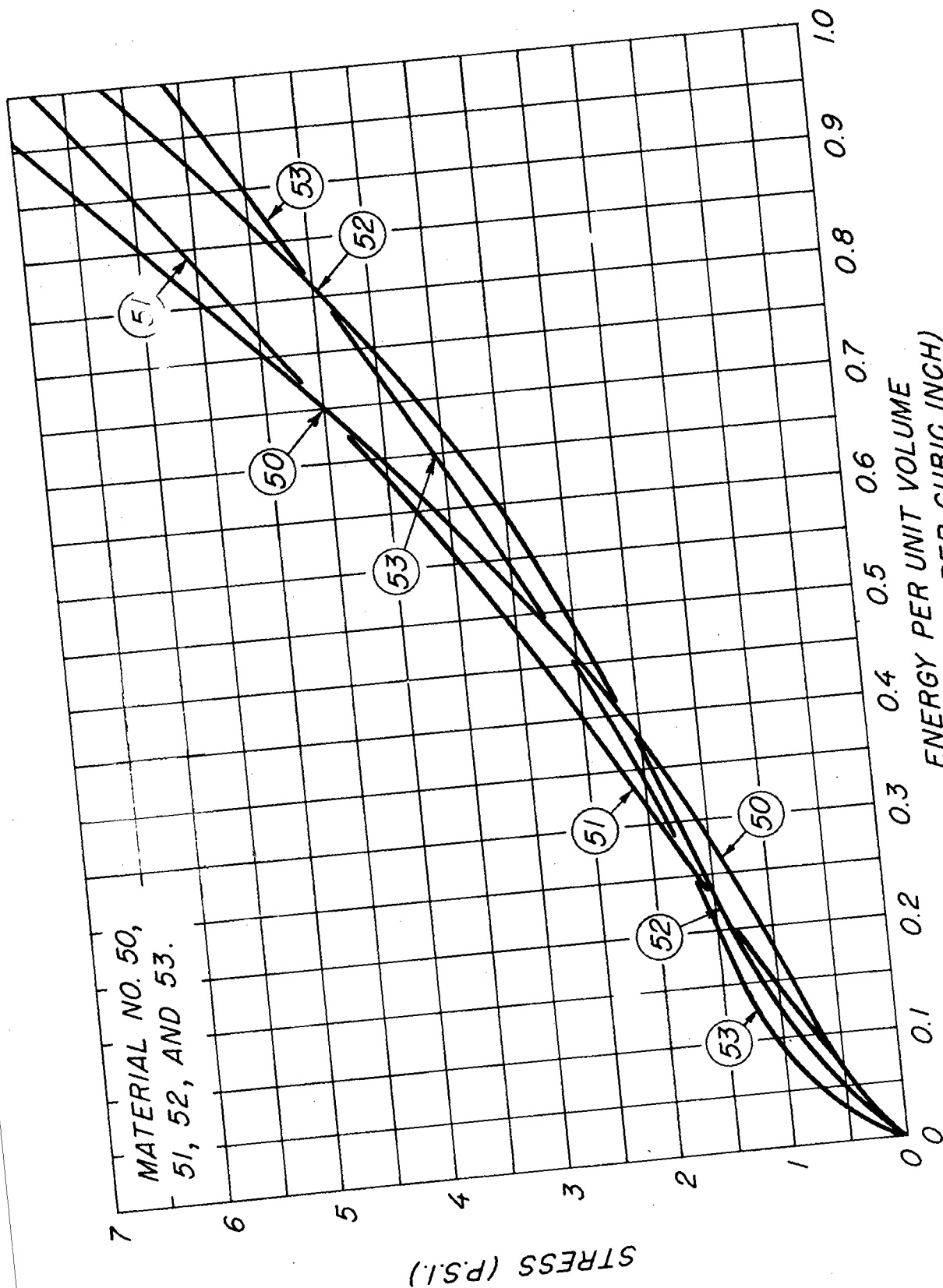


Figure 23.---Stress-energy per unit volume curves for materials 50, 51, 52, and 53.

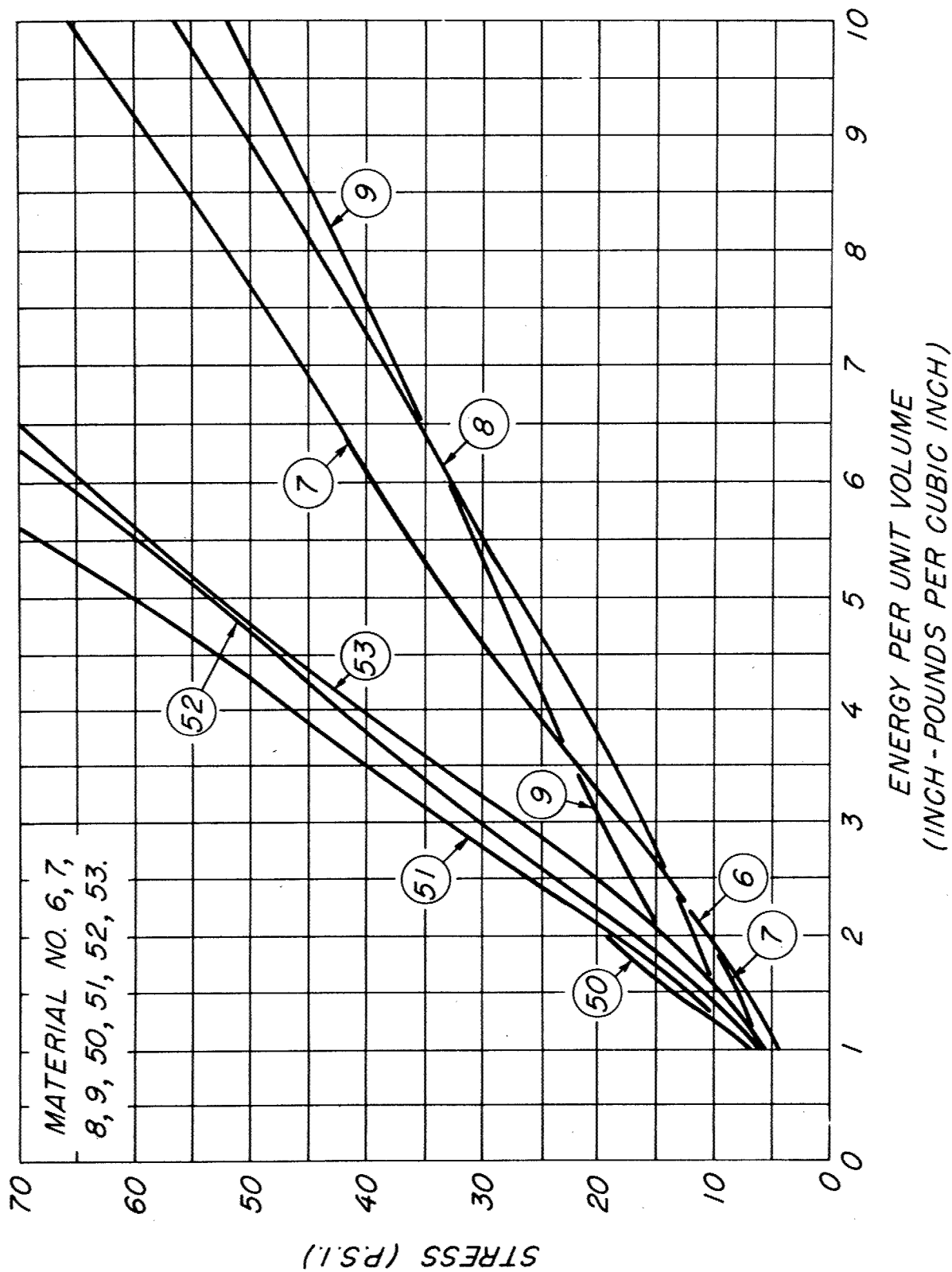


Figure 24.--Stress-energy per unit volume curves for materials 6, 7, 8, 9, 50, 51, 52, and 53.

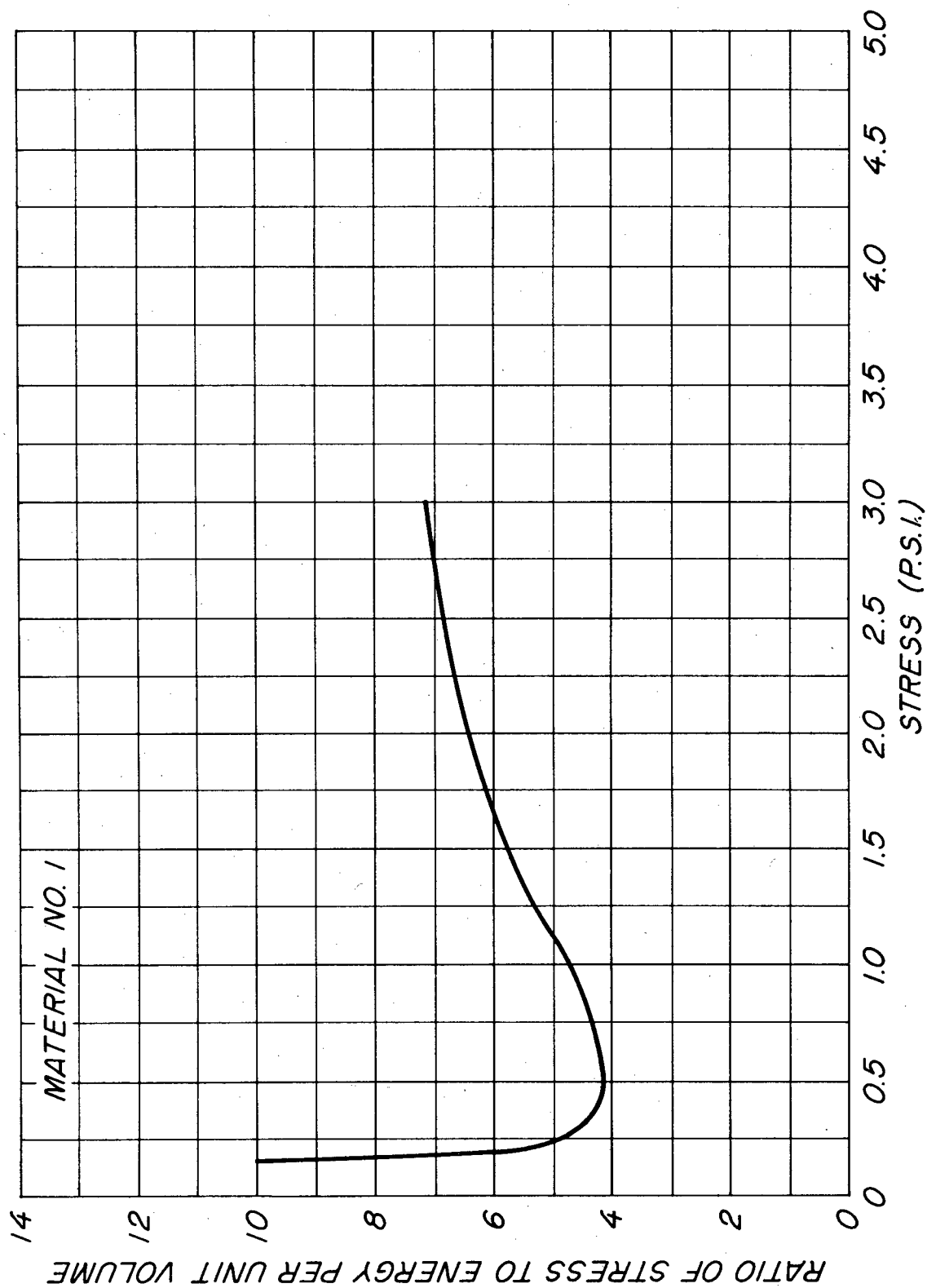


Figure 25.--A plot of the ratio of stress to energy per unit volume versus stress for material No. 1.

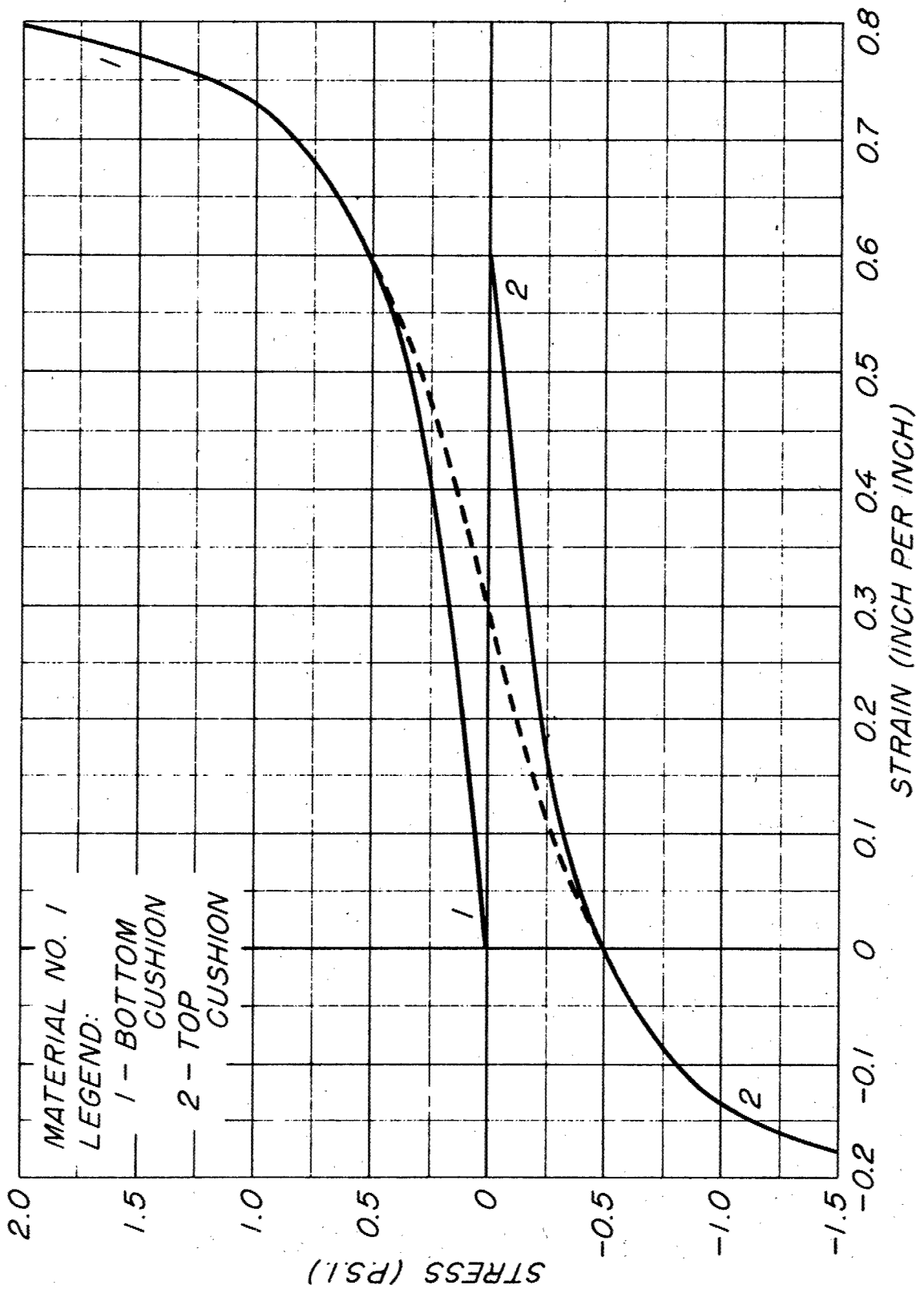


Figure 26.--Addition of stress-strain curves for material No. 1 compressed to a strain of 0.3.

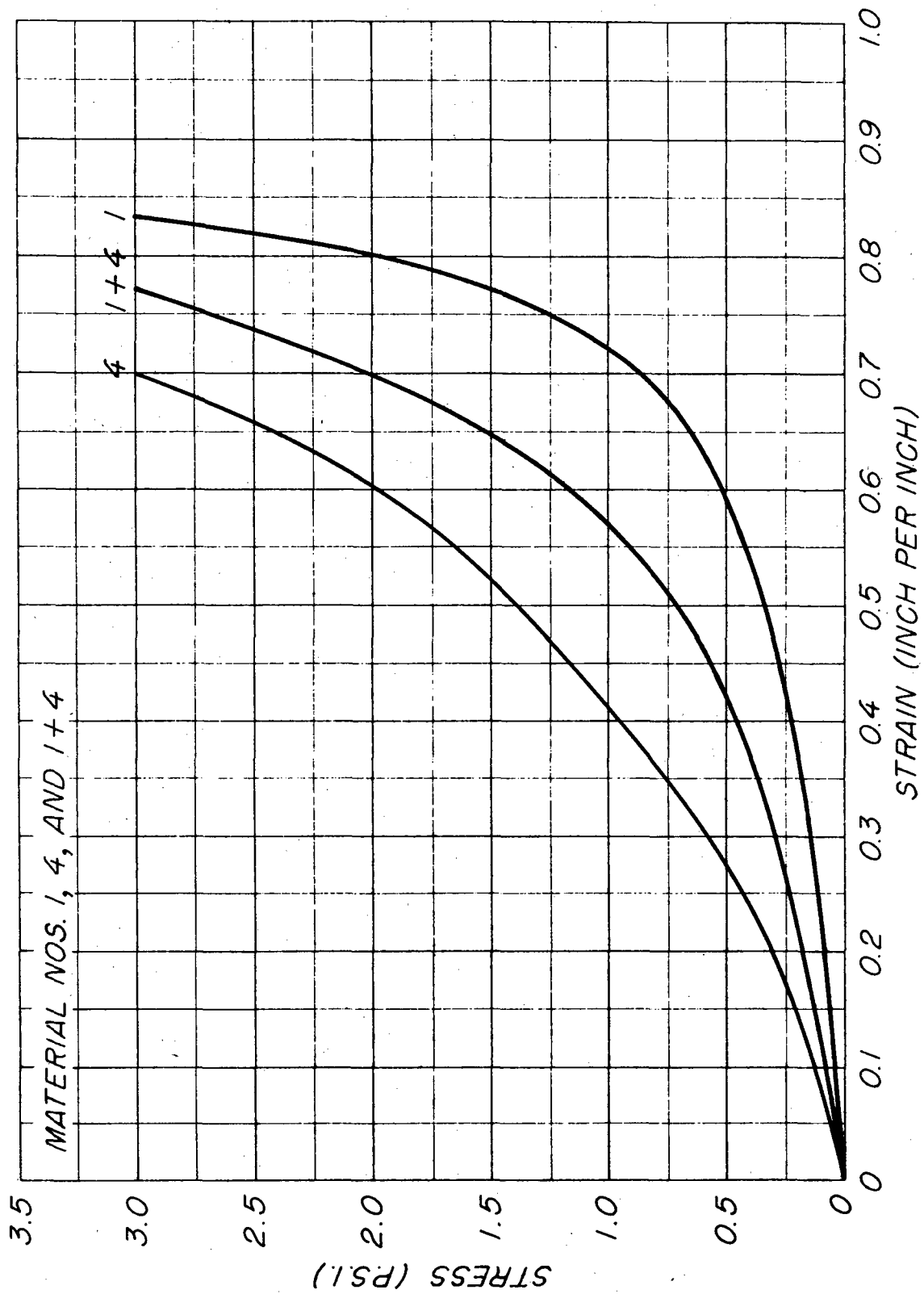


Figure 27.--Stress-strain curves for materials Nos. 1, 4, and equal thicknesses of materials 1 and 4 combined.

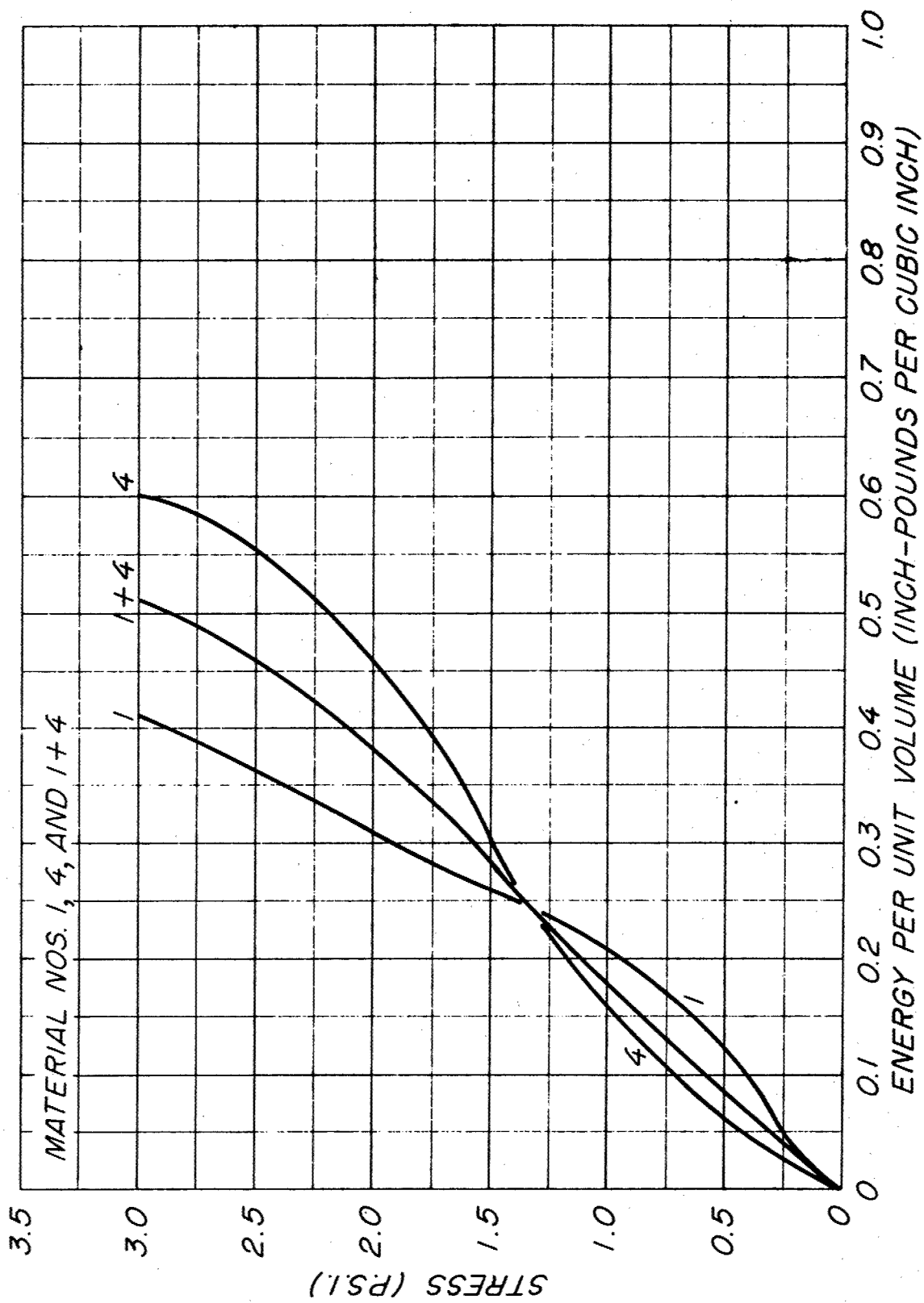


Figure 28.--Stress-energy per unit volume curves for materials Nos. 1, 4, and equal thicknesses of materials 1 and 4 combined.

APPENDIX I

Cushioning Materials Tested

Table 2.--Cushioning materials tested

Material No.	Description	Density based on dry weight	Thickness of samples tested	Moisture content at 75° F., 64 percent relative humidity
		<u>Lb. per cu. ft.</u>	<u>Inches</u>	<u>Percent</u>
1	Curled cattle hair bonded with natural latex or neoprene rubber	1.17 ₁ (0.94)	1.53 (2.95)	10.1 (9.7)
2do.....	1.56 (1.87)	1.43 (2.43)	8.7 (8.5)
3do.....	2.06 (2.15)	1.49 (2.47)	8.6 (8.5)
4do.....	2.62 (2.62)	1.35 (2.52)	7.3 (7.2)
5	Glass fibers bonded together with a resin	.80 (.99)	1.30 (1.50)	1.4 (less than 1)
6do.....	3.20 (2.90)	1.00 (1.50)	3.6 (1.2)
7do.....	5.00 (5.00)	1.00 (1.50)	2.6 (1.0)
8do.....	6.60 (6.93)	1.00 (1.54)	2.4 (1.0)
9do.....	8.40 (8.50)	1.00 (1.51)	Less than 1 (less than 1)
10	A lightweight, semirigid, synthetic composition embodying millions of tiny cells which are not connected	7.70	.53	Less than 1
11	Festooned sheet of curled cattle hair bonded with latex into a 1-inch-thick pad (one ply)	1.94	.91	10.9

(Sheet 1 of 4)

Table 2.--Cushioning materials tested (Continued)

Material No.	Description	Density based on dry weight	Thickness of samples tested	Moisture content at 75° F., 64 percent relative humidity
		<u>Lb. per cu. ft.</u>	<u>Inches</u>	<u>Percent</u>
12	Two plies of material No. 11 cross-laminated with latex binder	1.87	1.66	10.7
13	Three plies of material No. 11 cross-laminated with latex binder	2.03	2.51	10.9
14	Creped wadding, dimple embossed, brown in color	2.93	.67	8.9
15	Creped wadding, K embos- sed, white in color	2.98	.60	8.4
16	Felted wool	10.55	.49	14.3
17do.....	9.10	.63	13.6
18do.....	9.10	.54	13.9
19	Rubberized wool	2.87	.94	5.3
20do.....	3.25	1.13	4.8
21do.....	4.07	1.22	4.5
22	Rubberized cattle hair	1.16	2.59	7.4
23do.....	1.48	2.00	5.3
24do.....	1.75	1.42	5.6
25	Wood-fiber felt	1.46	1.02	10.0
26do.....	2.22	1.13	8.6
27do.....	2.90	1.31	
28	Closed cellular synthetic rubber	11.75	.95	2.5

Table 2.--Cushioning materials tested (Continued)

Material No.	Description	Density based on dry weight	Thickness of samples tested	Moisture content at 75° F., 64 percent relative humidity
		<u>Lb. per cu. ft.</u>	<u>Inches</u>	<u>Percent</u>
29	Glass fibers bonded together	0.61	1.23	2.32
30do.....	.83	1.23	2.02
31	Curled cattle hair bonded together with latex	1.86	1.35	7.94
32do.....	2.06	2.18	7.98
33do.....	1.96	2.27	7.25
34do.....	2.03	3.54	7.16
35	Sponge rubber	6.19	.88	Less than 1
36do.....	7.70	.78	Less than 1
37do.....	8.42	.92	Less than 1
38	Glass fibers bonded together	.72	.90	2.64
39do.....	.76	.89	1.96
40do.....	.88	1.08	1.33
41do.....	1.28	1.18	1.34
42do.....	1.42	1.25	1.03
43do.....	1.98	1.45	1.20
44	Cotton wadding, surface sizing of starch or resin, white color	2.00	.49	7.9
45do.....	2.29	.62	8.0

Table 2.--Cushioning materials tested (Continued)

Material No.	Description	Density	Thickness	Moisture content
		based on dry weight	of samples tested	at 75° F., 64 percent relative humidity
		<u>Lb. per cu. ft.</u>	<u>Inches</u>	<u>Percent</u>
48	Cotton wadding, surface sizing of starch or resin, blue color	2.55	0.36	10.2
49do.....	2.33	.78	10.7
50	Reclaimed sponge rubber; ground and bonded together	7.70	1.06	Less than 1
51do.....	8.95	1.12	Less than 1
52do.....	10.36	1.12	Less than 1
53do.....	12.00	1.11	Less than 1

¹Numbers in parentheses indicate additional thicknesses tested but not reported because of close agreement with data reported.

(Sheet 4 of 4)

APPENDIX II

Stress-strain and Energy Per Unit Volume-strain

Curves for Various Cushioning Materials

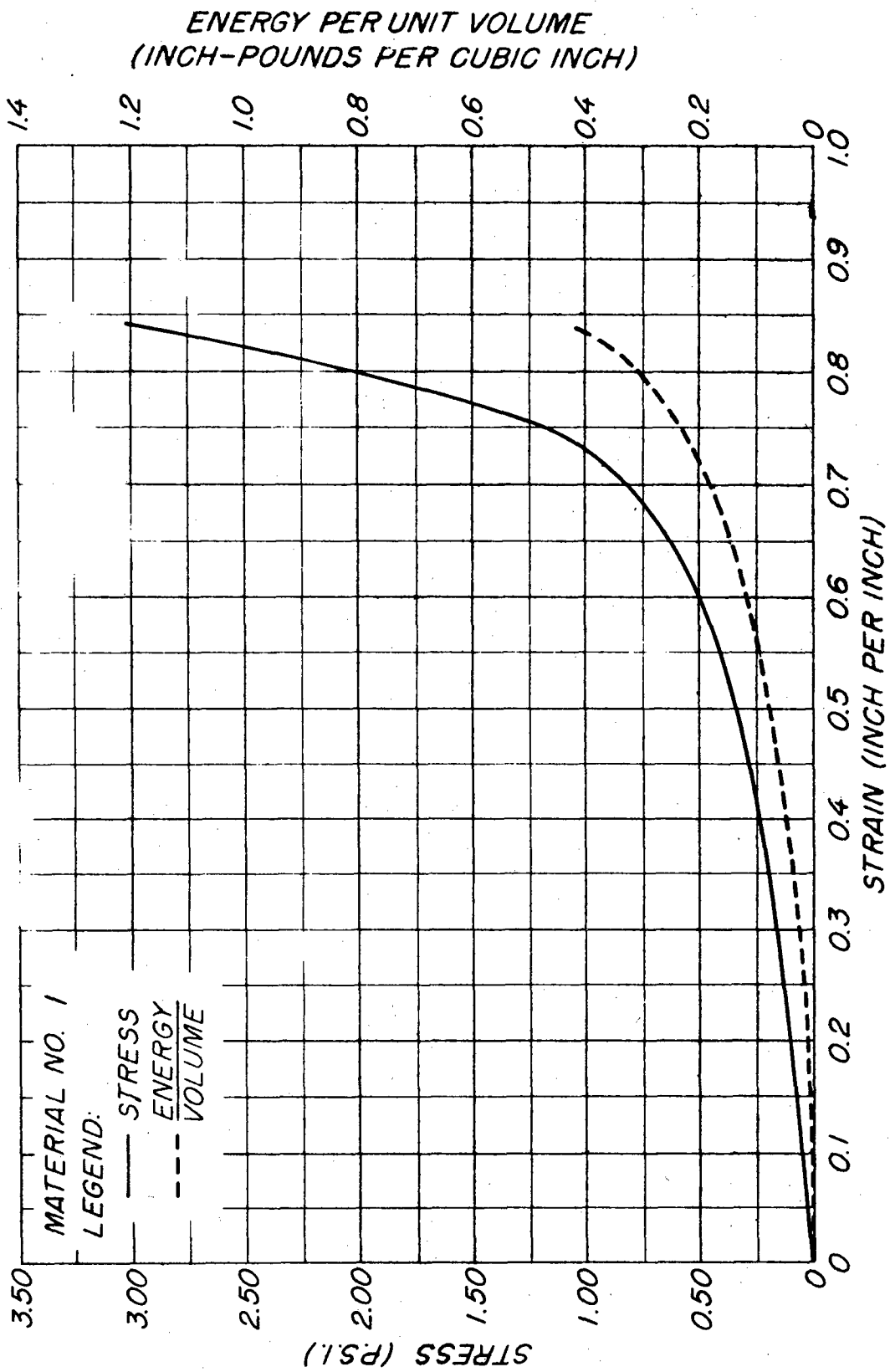


Figure 29.--Stress-strain and energy per unit volume-strain curves for material No. 1.

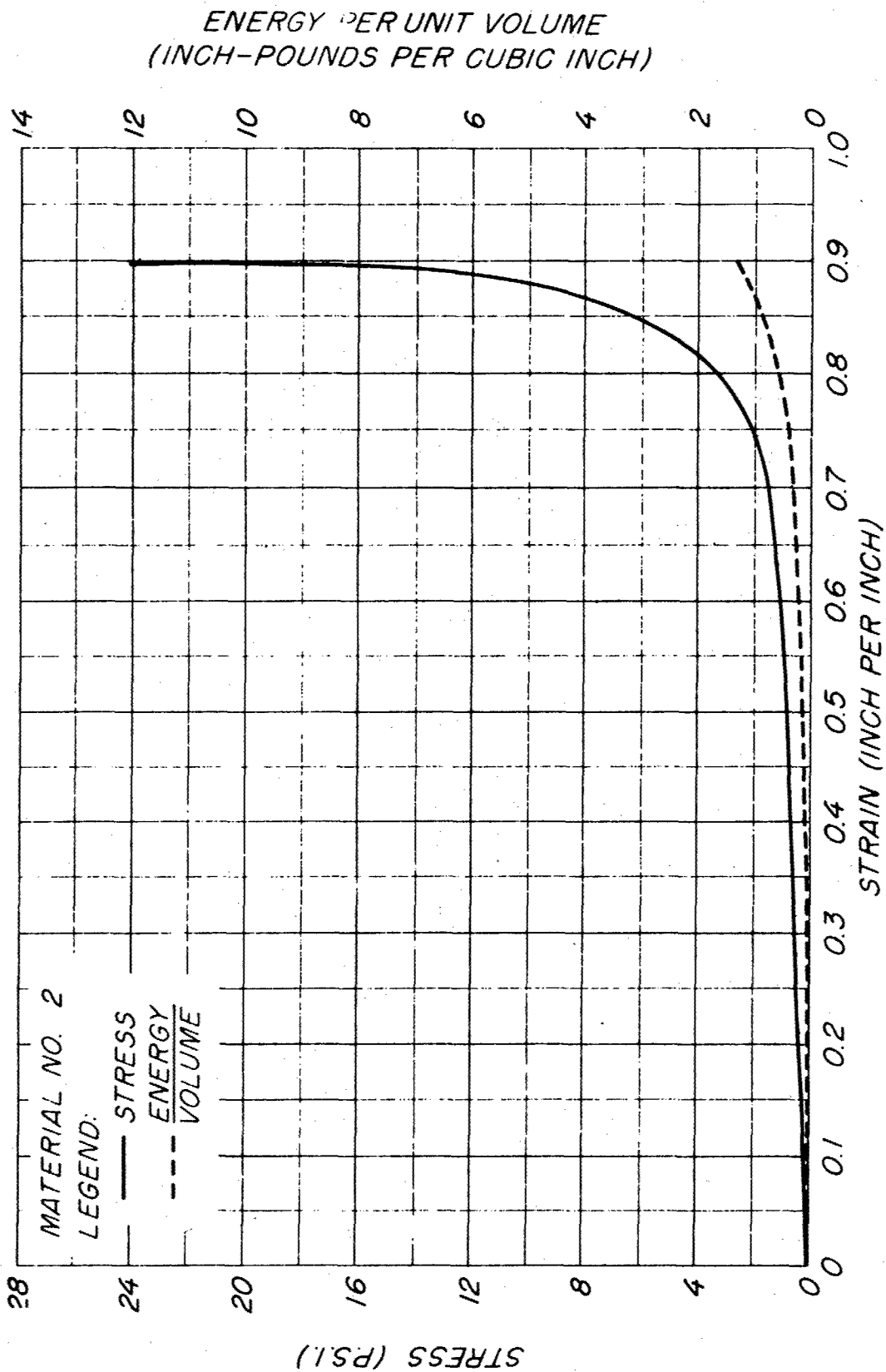


Figure 30.---Stress-strain and energy per unit volume-strain curves for material No. 2
(Stress 0-28 P.S.I.).

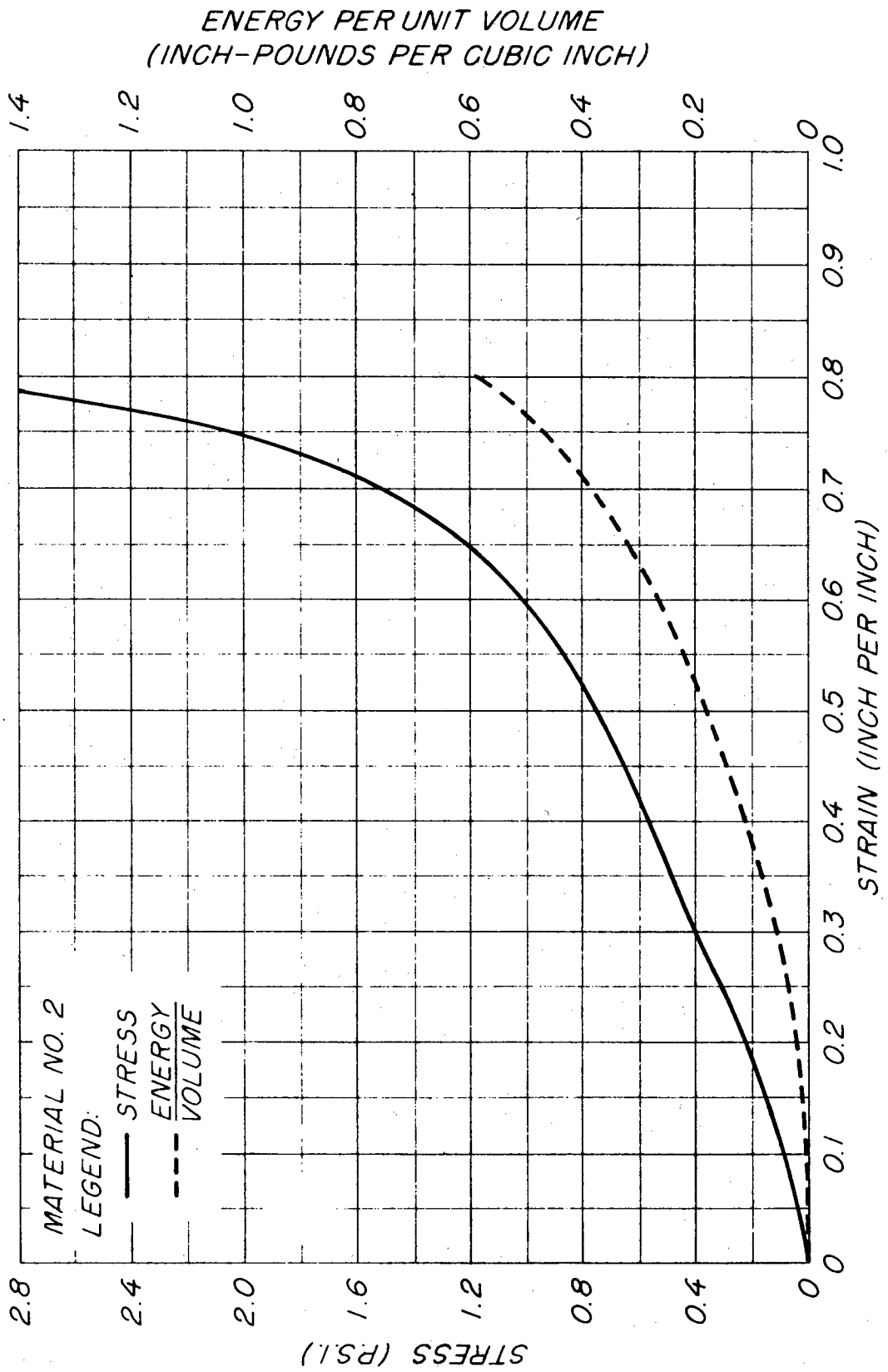


Figure 31.--Stress-strain and energy per unit volume-strain curves for material No. 2
(Stress 0-2.8 P.S.I.).

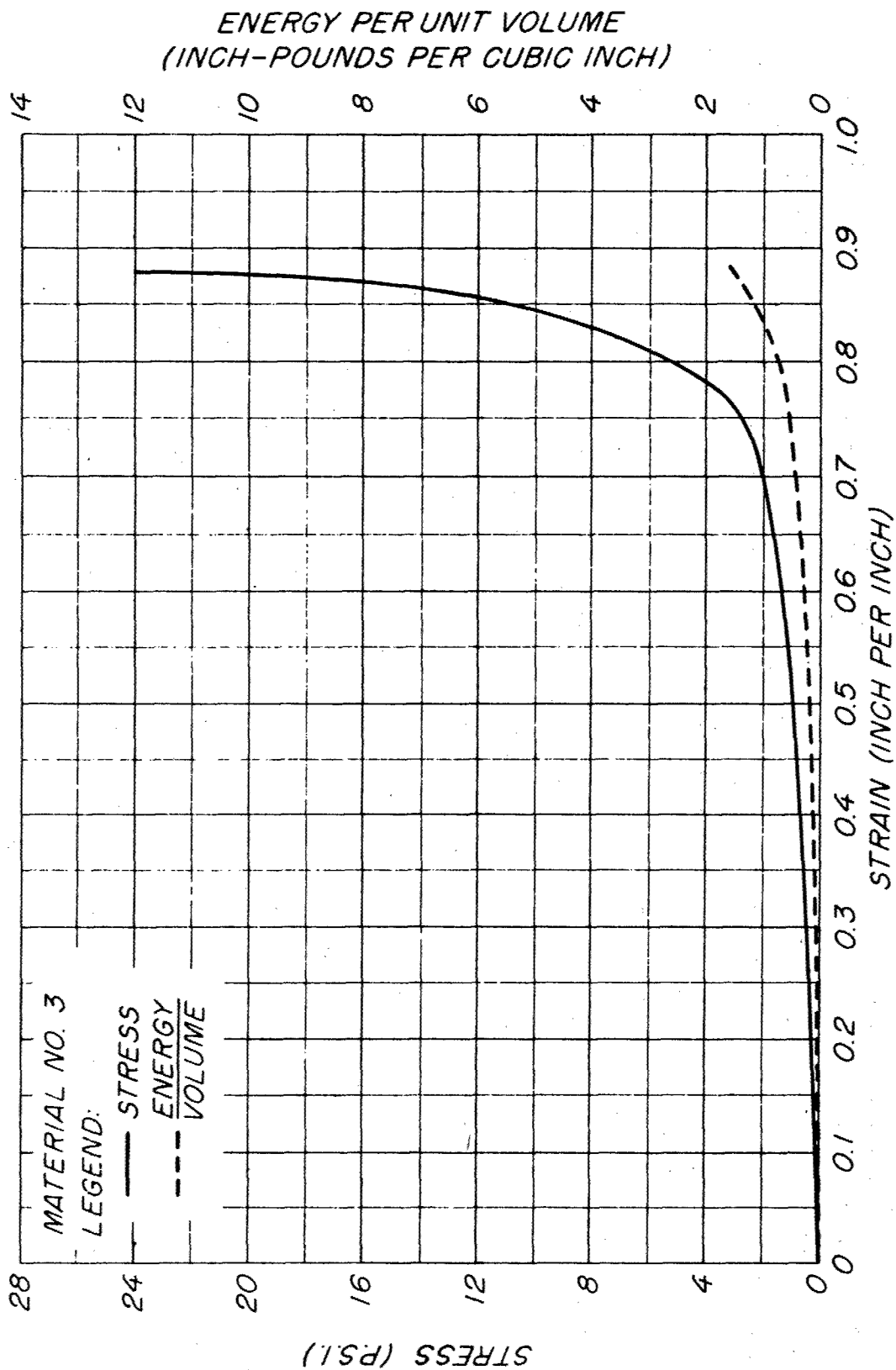


Figure 32.--Stress-strain and energy per unit volume-strain curves for material No. 3
(Stress 0-28 P.S.I.).

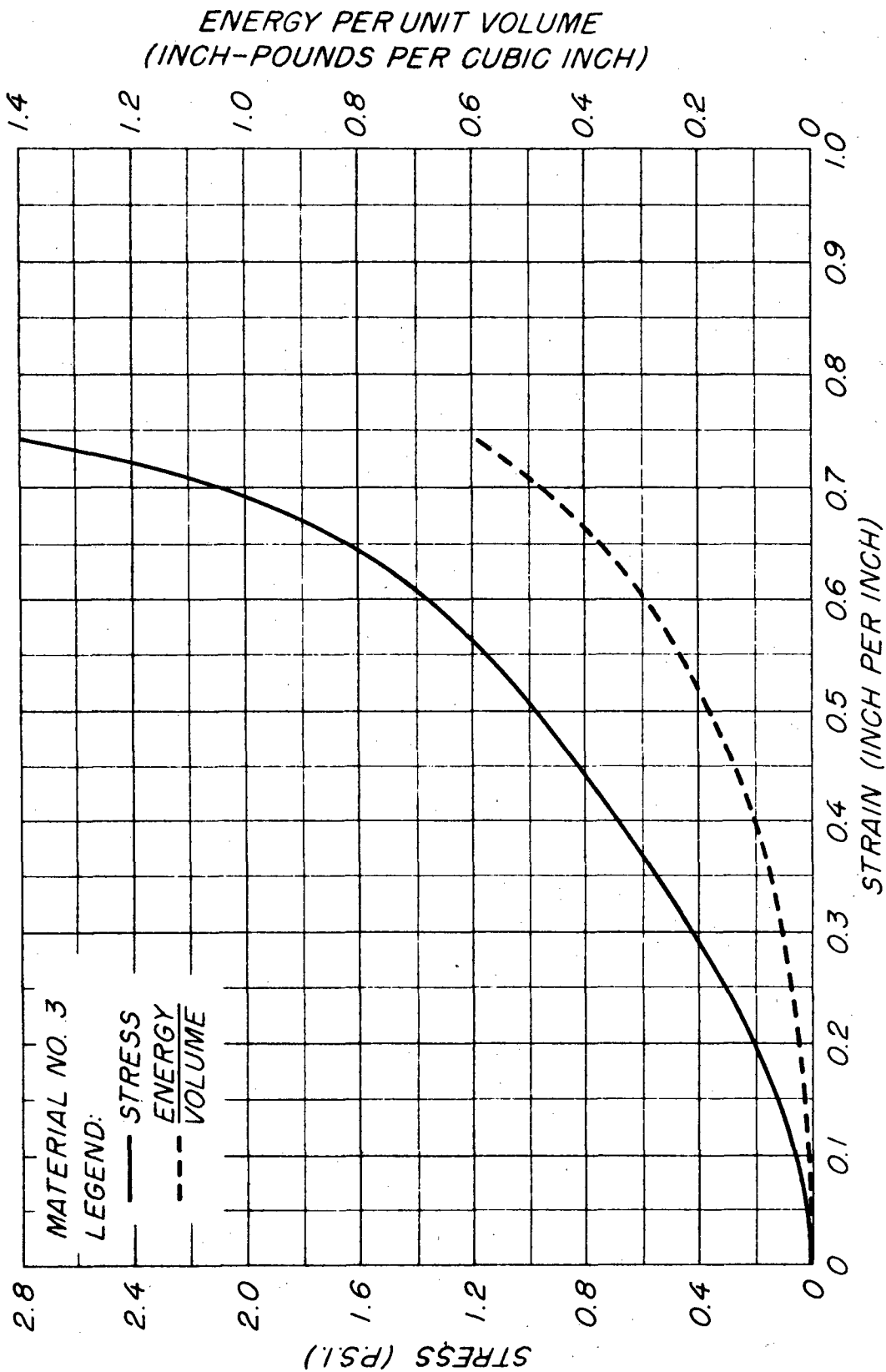


Figure 33.--Stress-strain and energy per unit volume-strain curves for material No. 3
(Stress 0-2.8 P.S.I.).

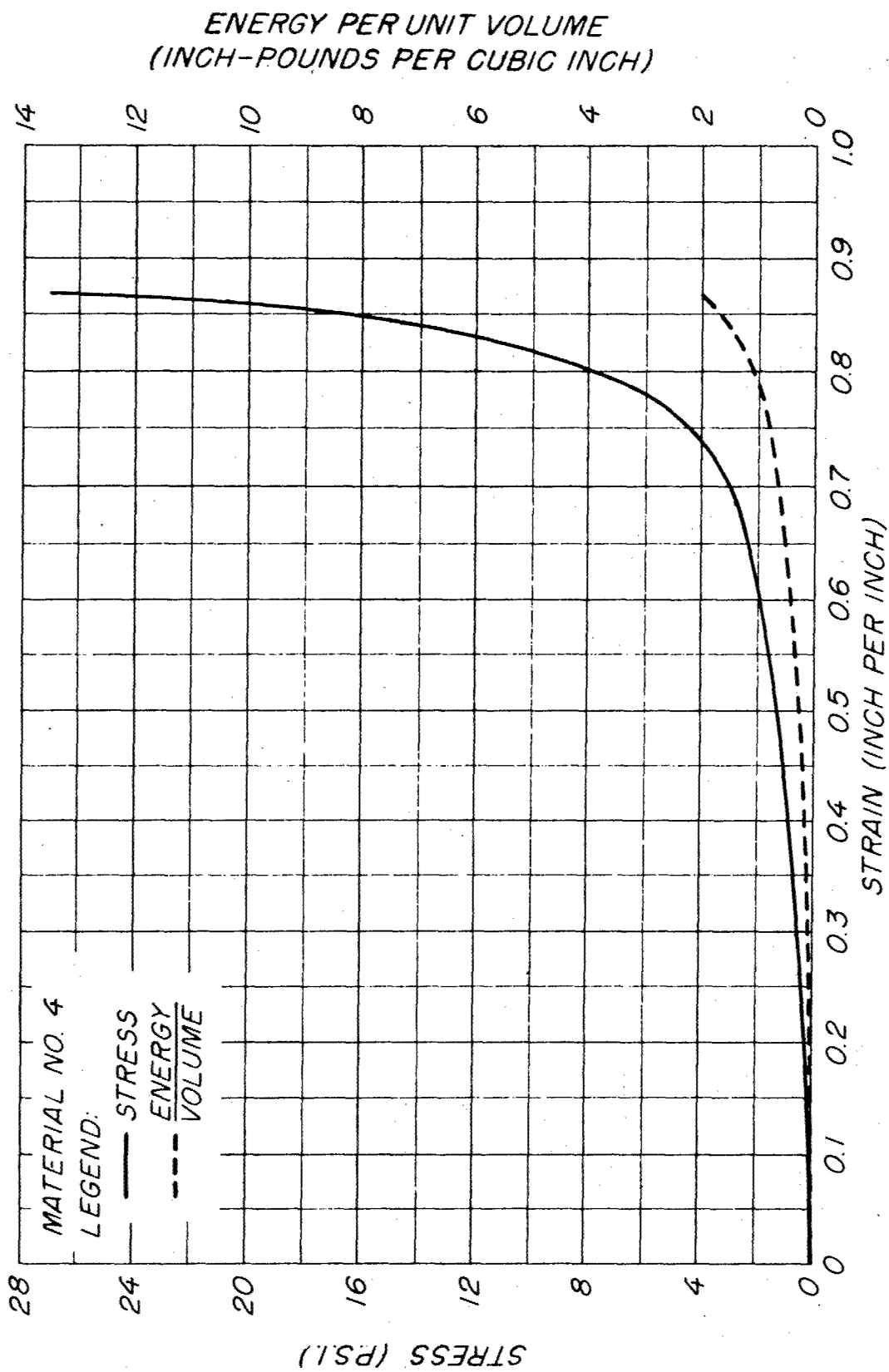


Figure 34.--Stress-strain and energy per unit volume-strain curves for material No. 4
(Stress 0-28 P.S.I.).

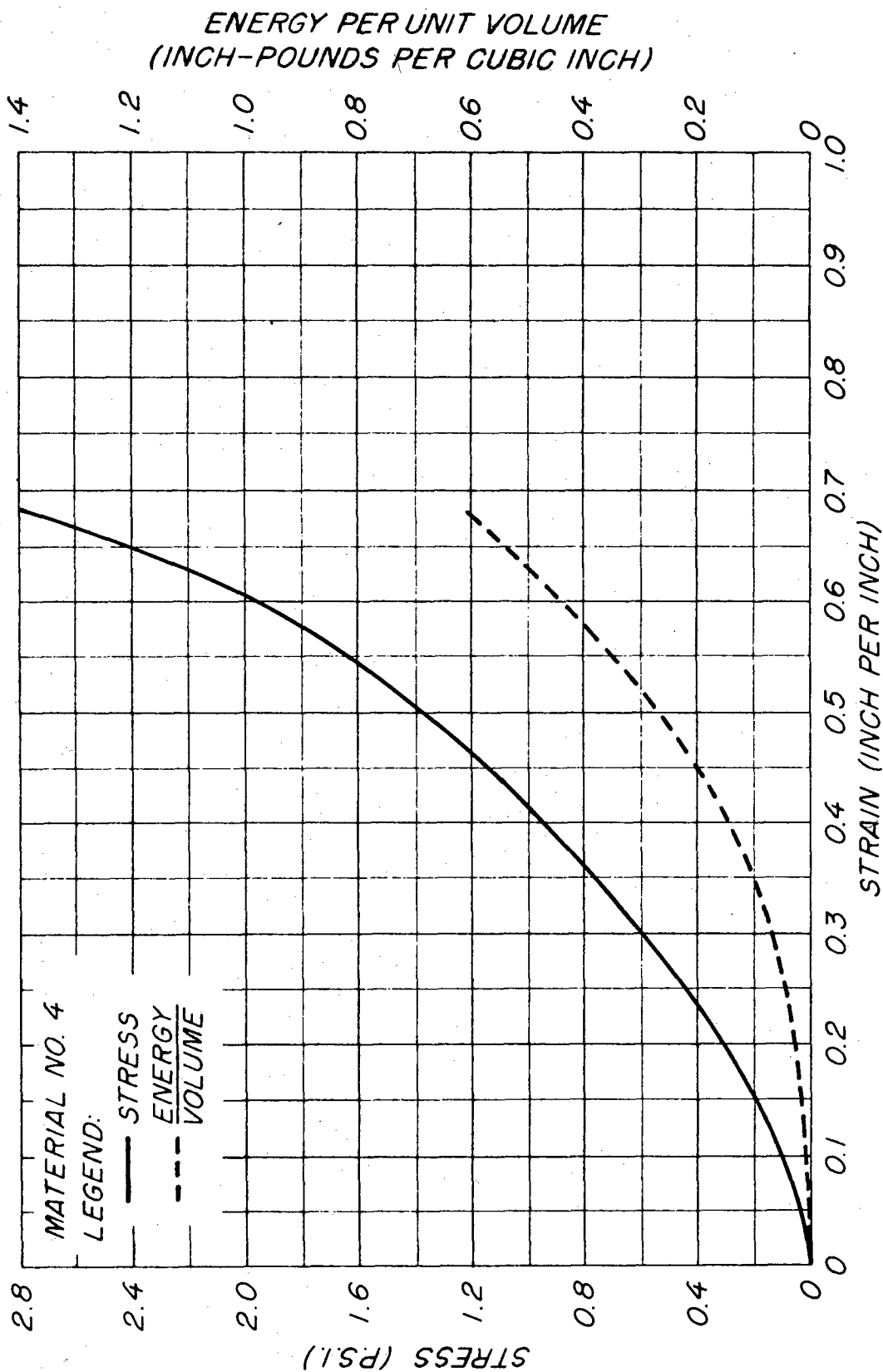


Figure 35.--Stress-strain and energy per unit volume-strain curves for material No. 4
(Stress 0-2.8 P.S.I.).

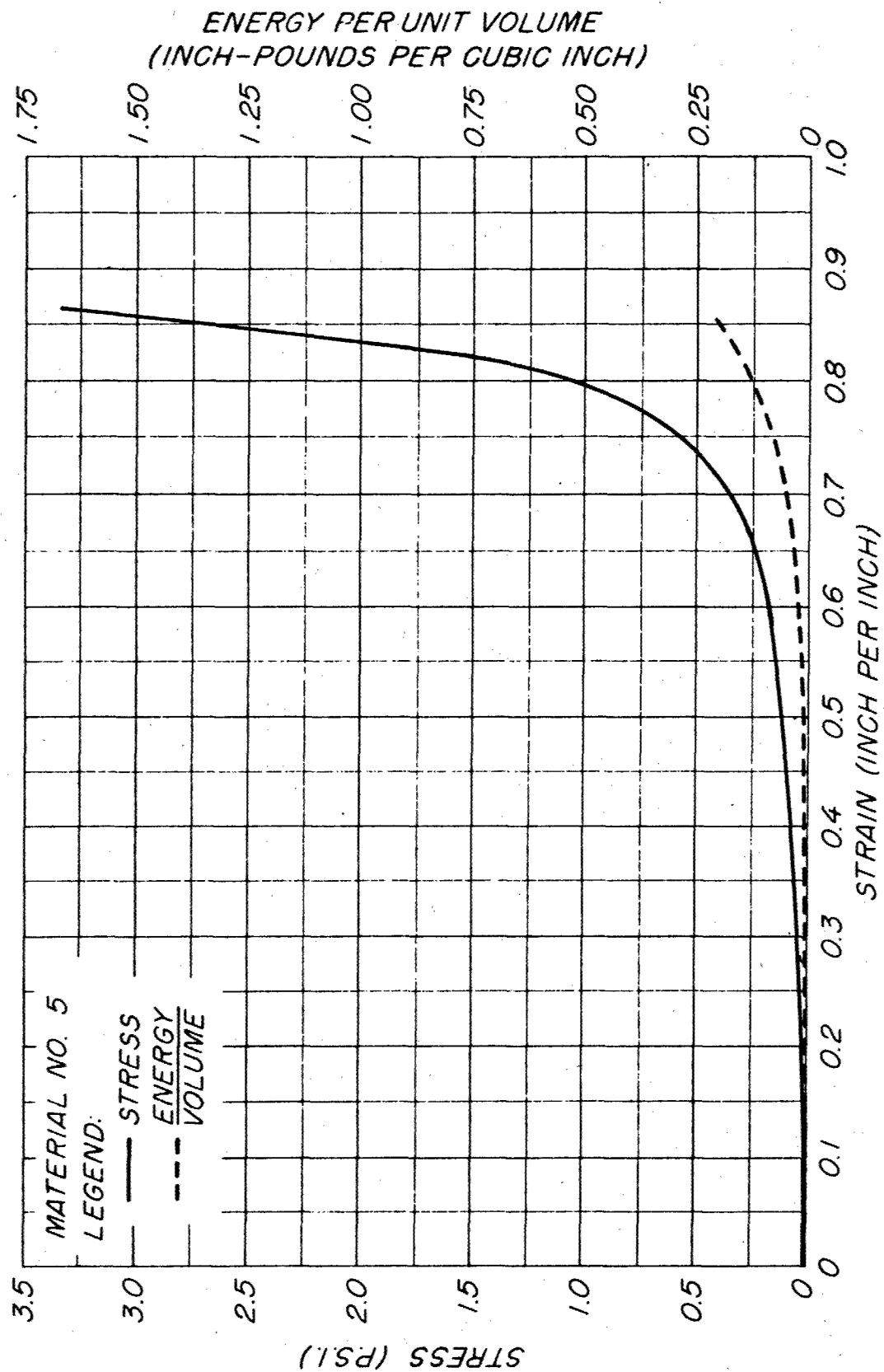


Figure 36.--Stress-strain and energy per unit volume-strain curves for material No. 5
(Stress 0-3.5 P.S.I.).

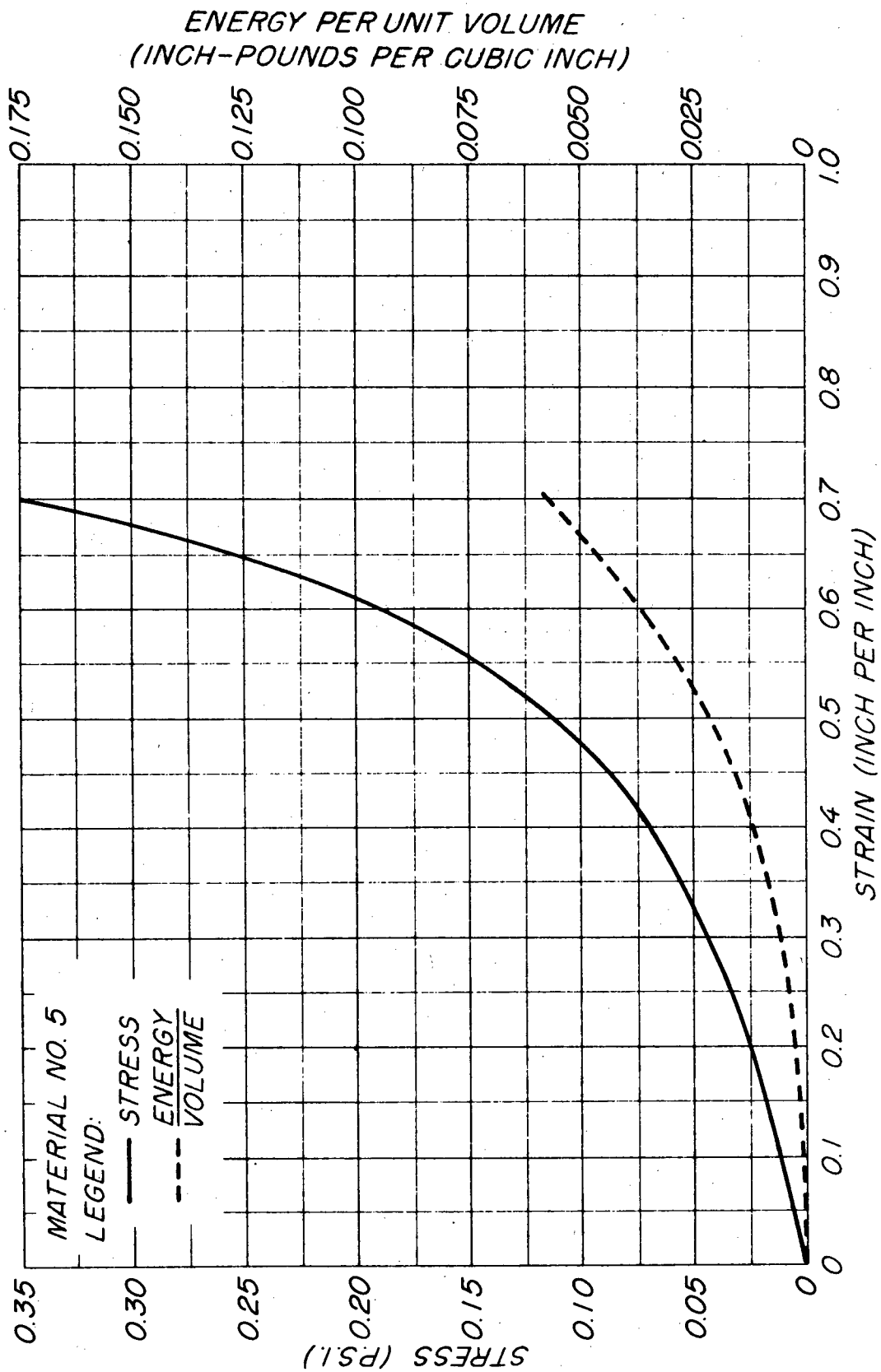


Figure 37.--Stress-strain and energy per unit volume-strain curves for material No. 5
(Stress 0-0.35 P.S.I.).

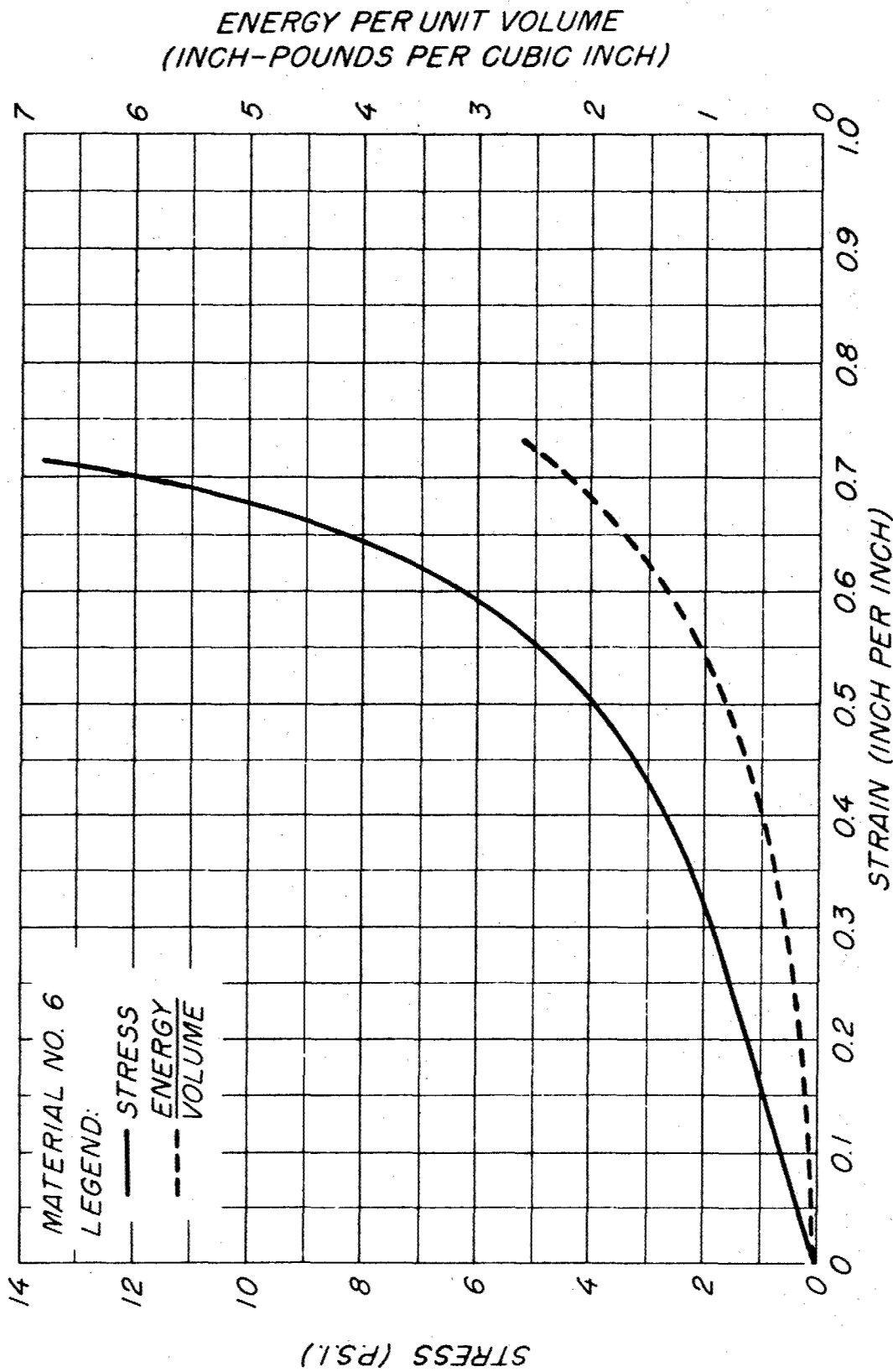


Figure 38.--Stress-strain and energy per unit volume-strain curves for material No. 6.

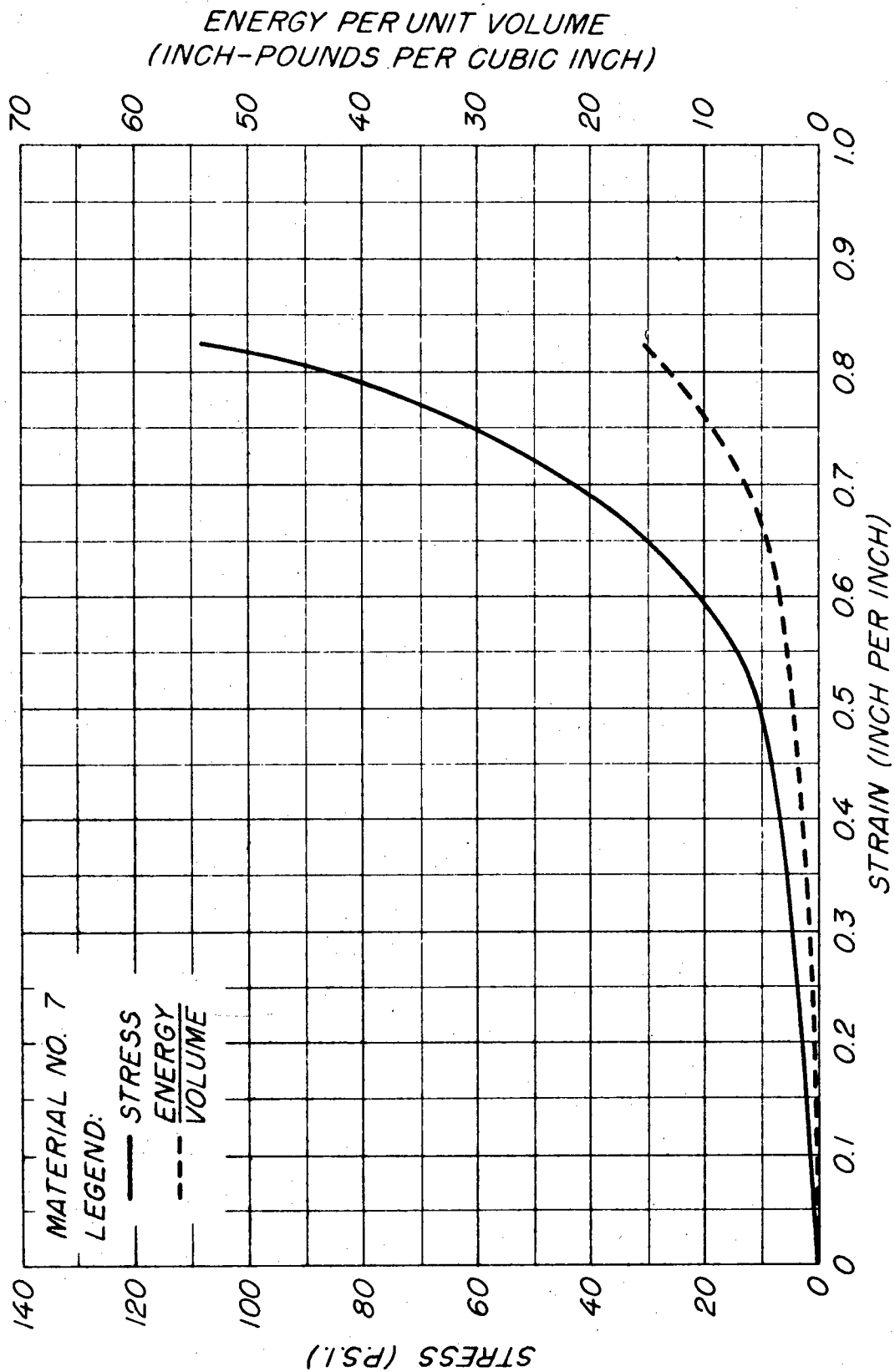


Figure 39.---Stress-strain and energy per unit volume-strain curves for material No. 7
(Stress 0-140 P.S.I.).

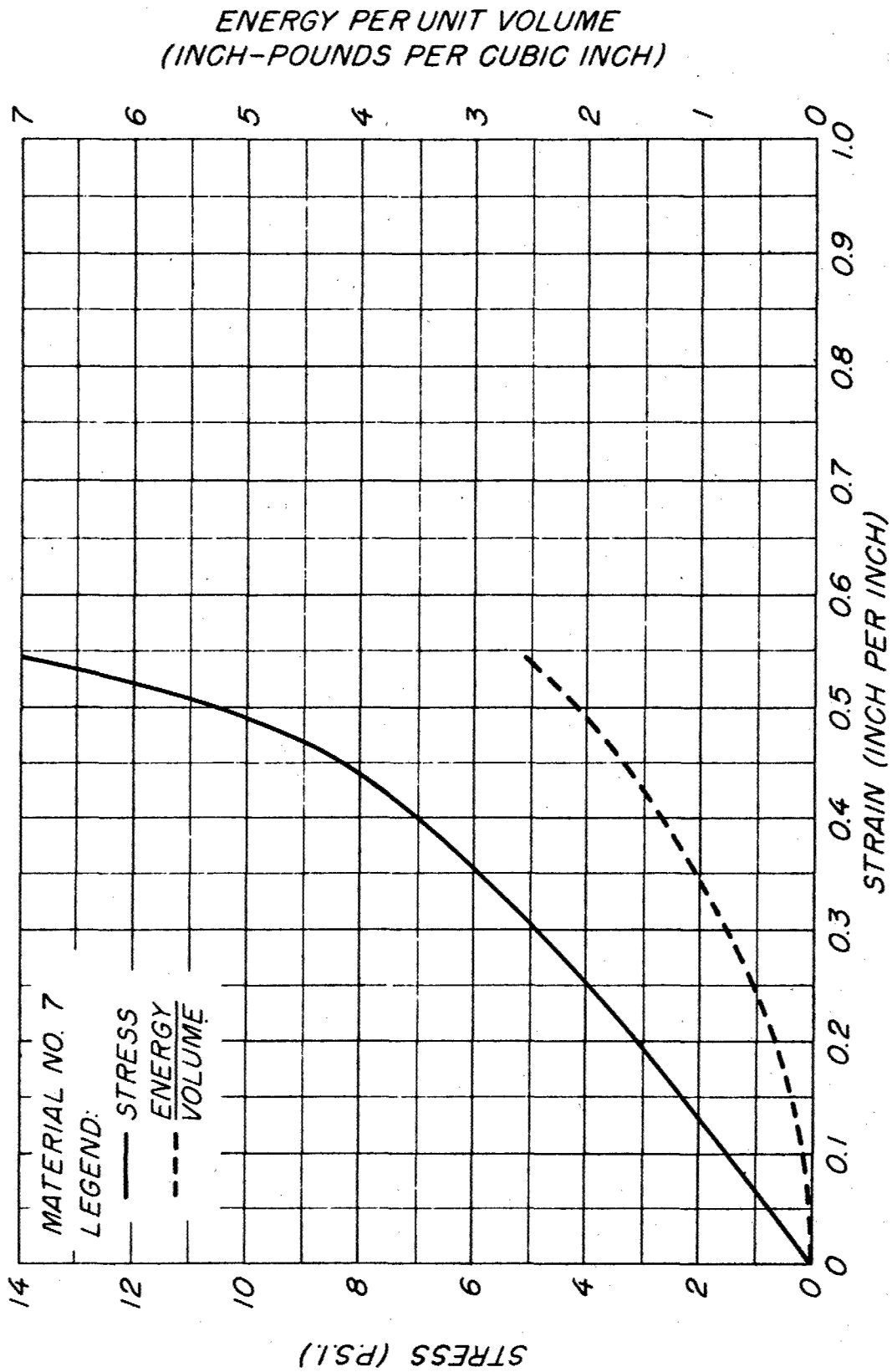


Figure 40.---Stress-strain and energy per unit volume-strain curves for material No. 7
(Stress 0-14 P.S.I.).

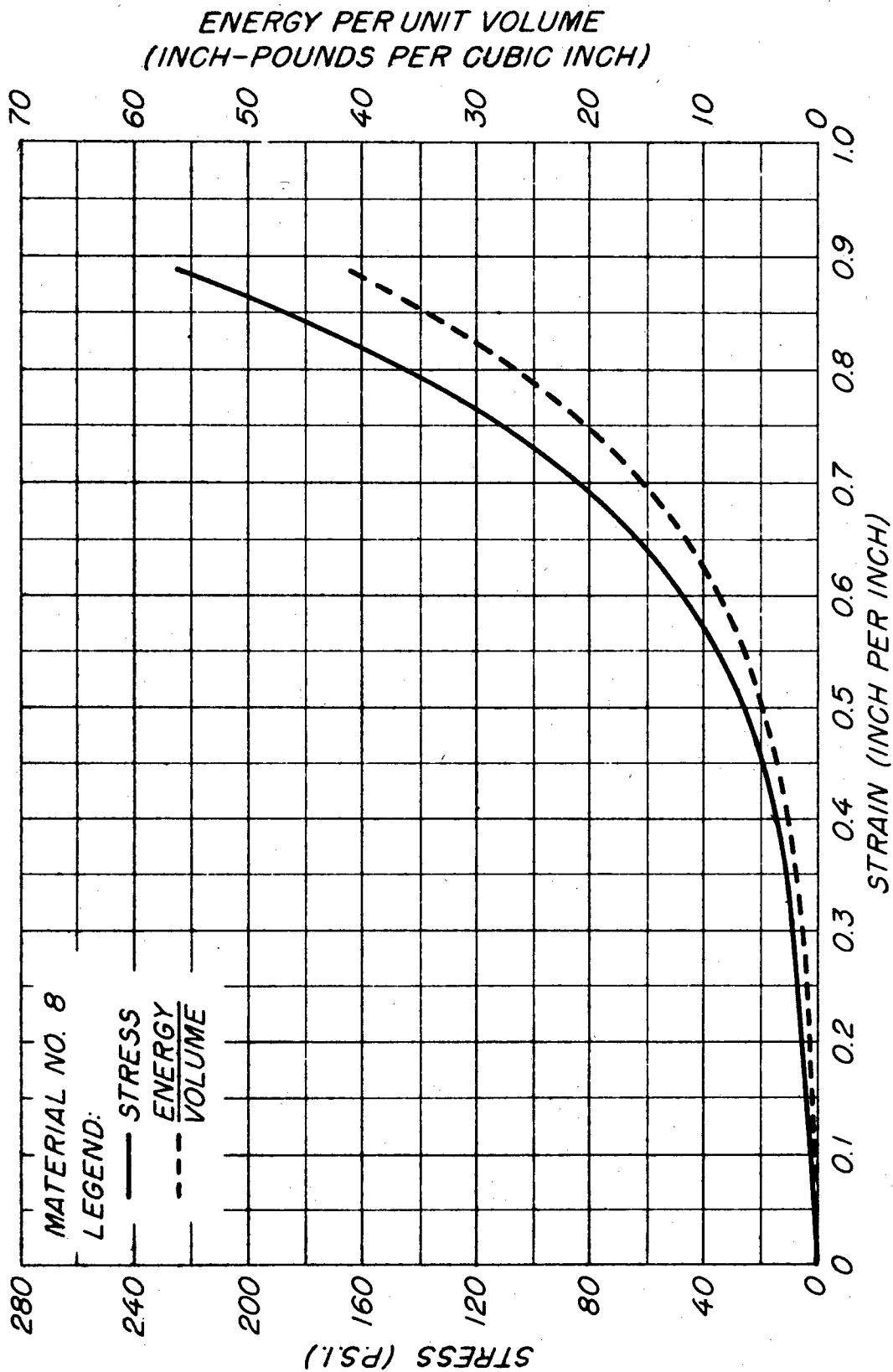


Figure 41.--Stress-strain and energy per unit volume-strain curves for material No. 8
(Stress 0-280 P.S.I.).

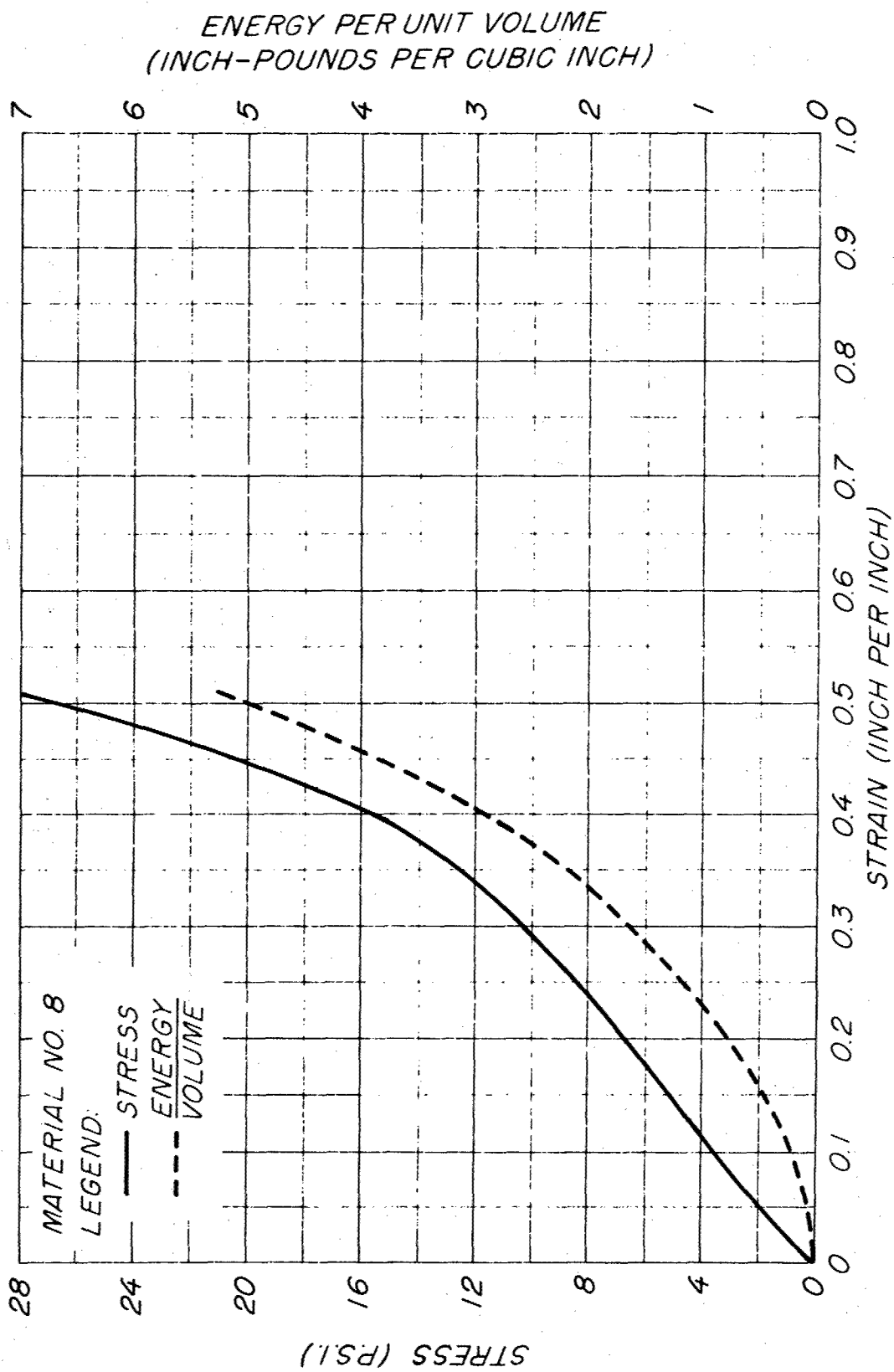


Figure 42.--Stress-strain and energy per unit volume-strain curves for material No. 8
(Stress C-28 P.S.I.).

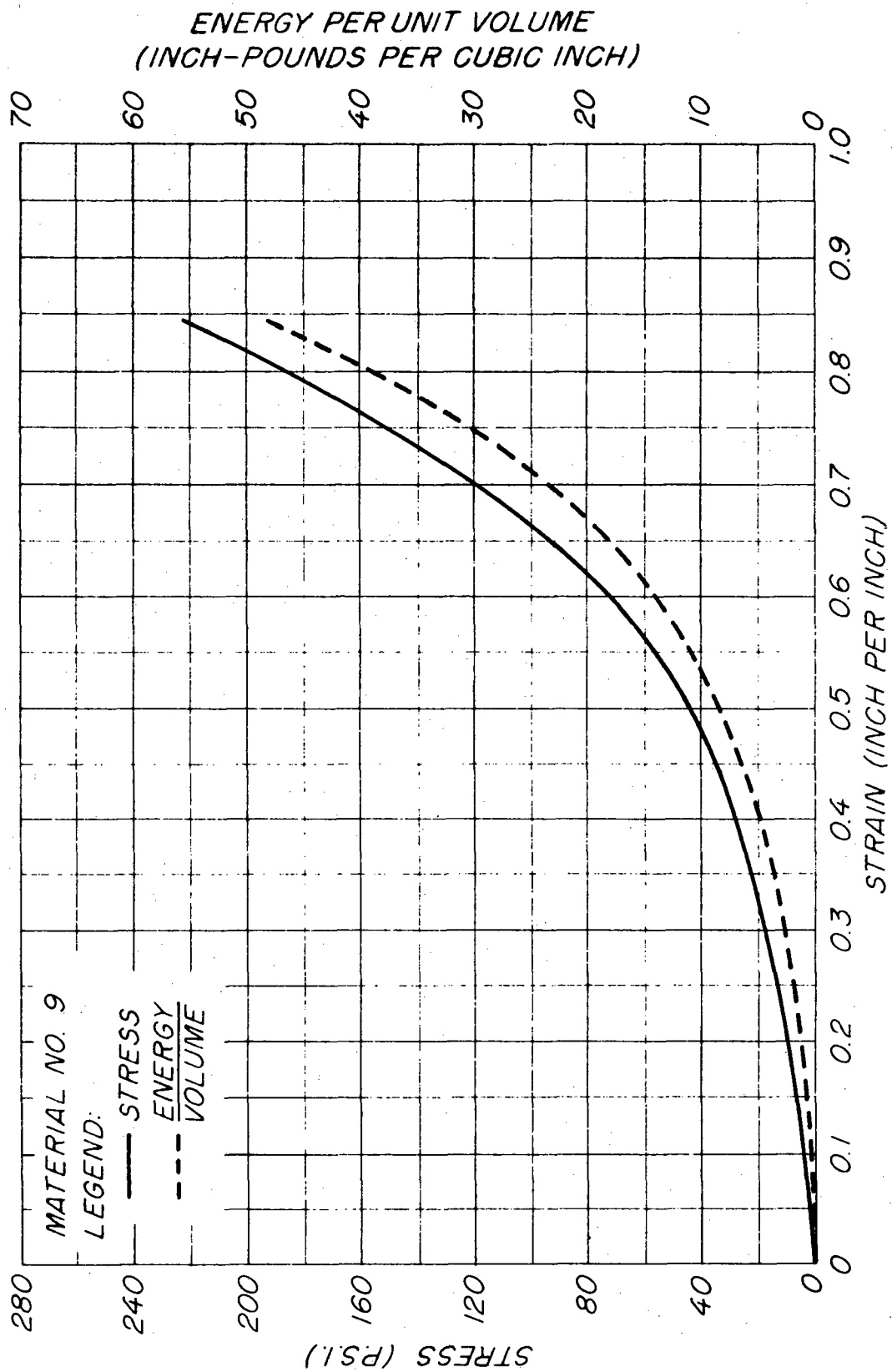


Figure 43.--Stress-strain and energy per unit volume-strain curves for material No. 9.

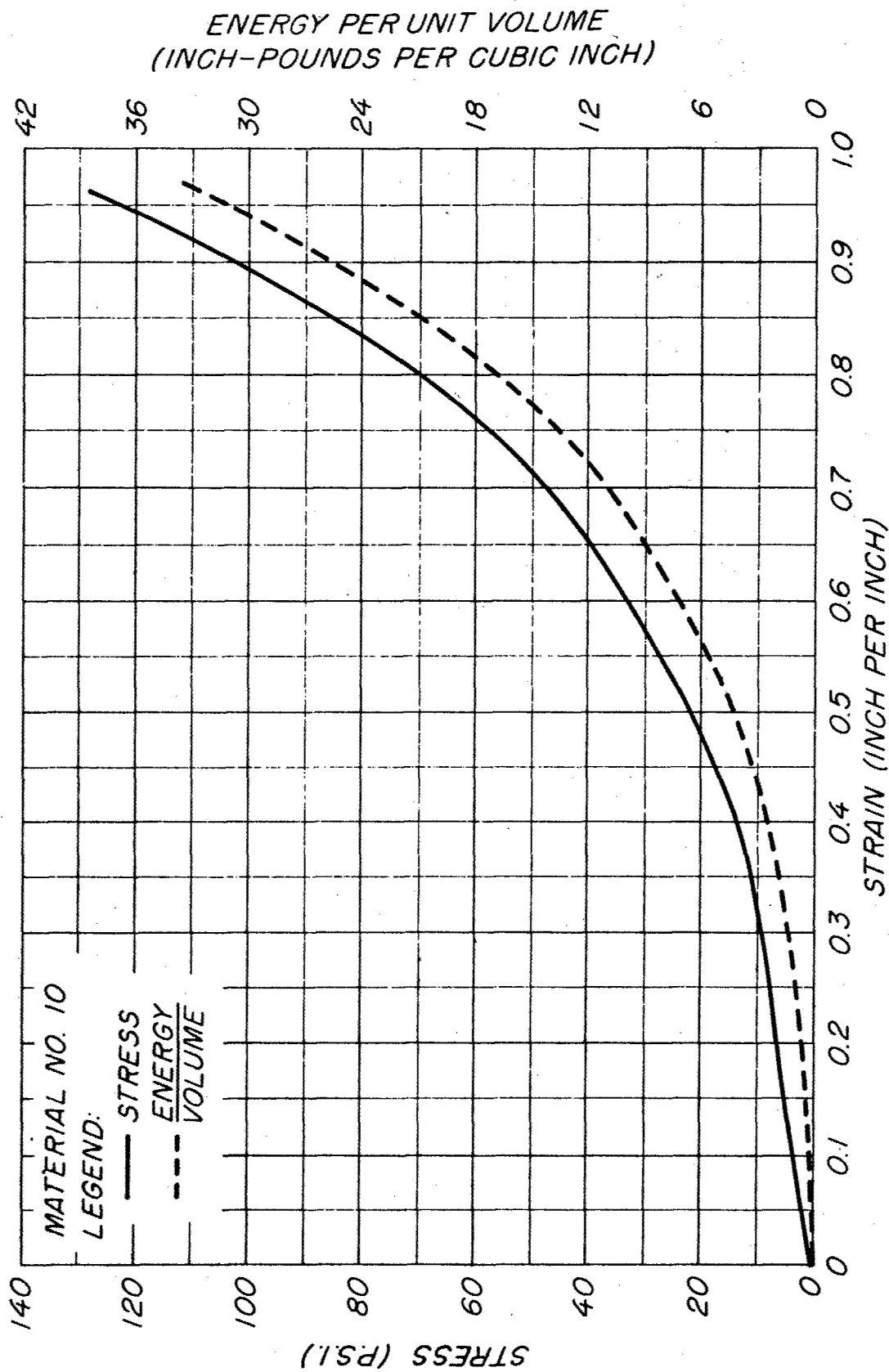


Figure 44.--Stress-strain and energy per unit volume-strain curves for material No. 10.

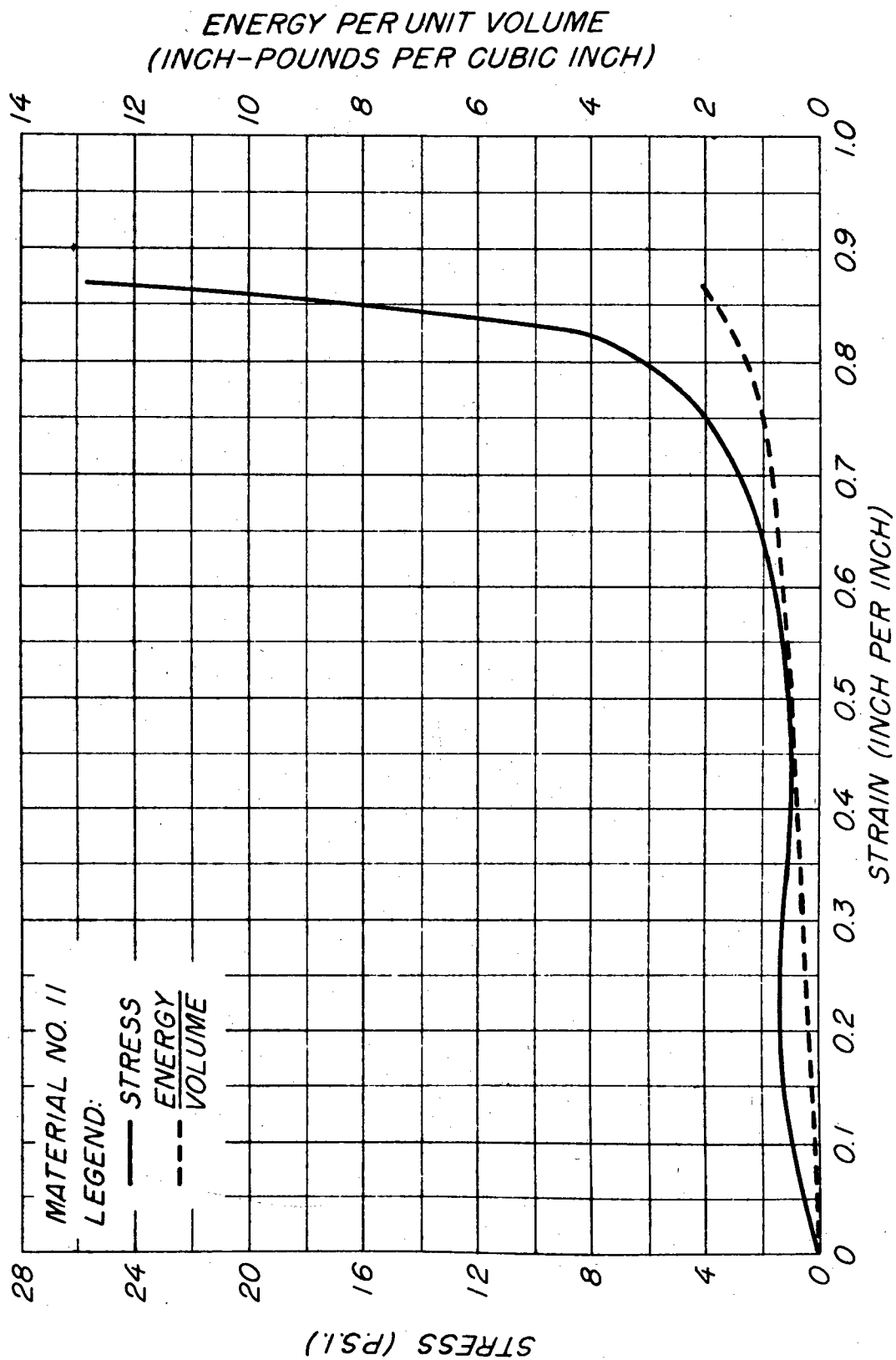


Figure 45.---Stress-strain and energy per unit volume-strain curves for material No. 11
(Stress 0-28 P.S.I.).

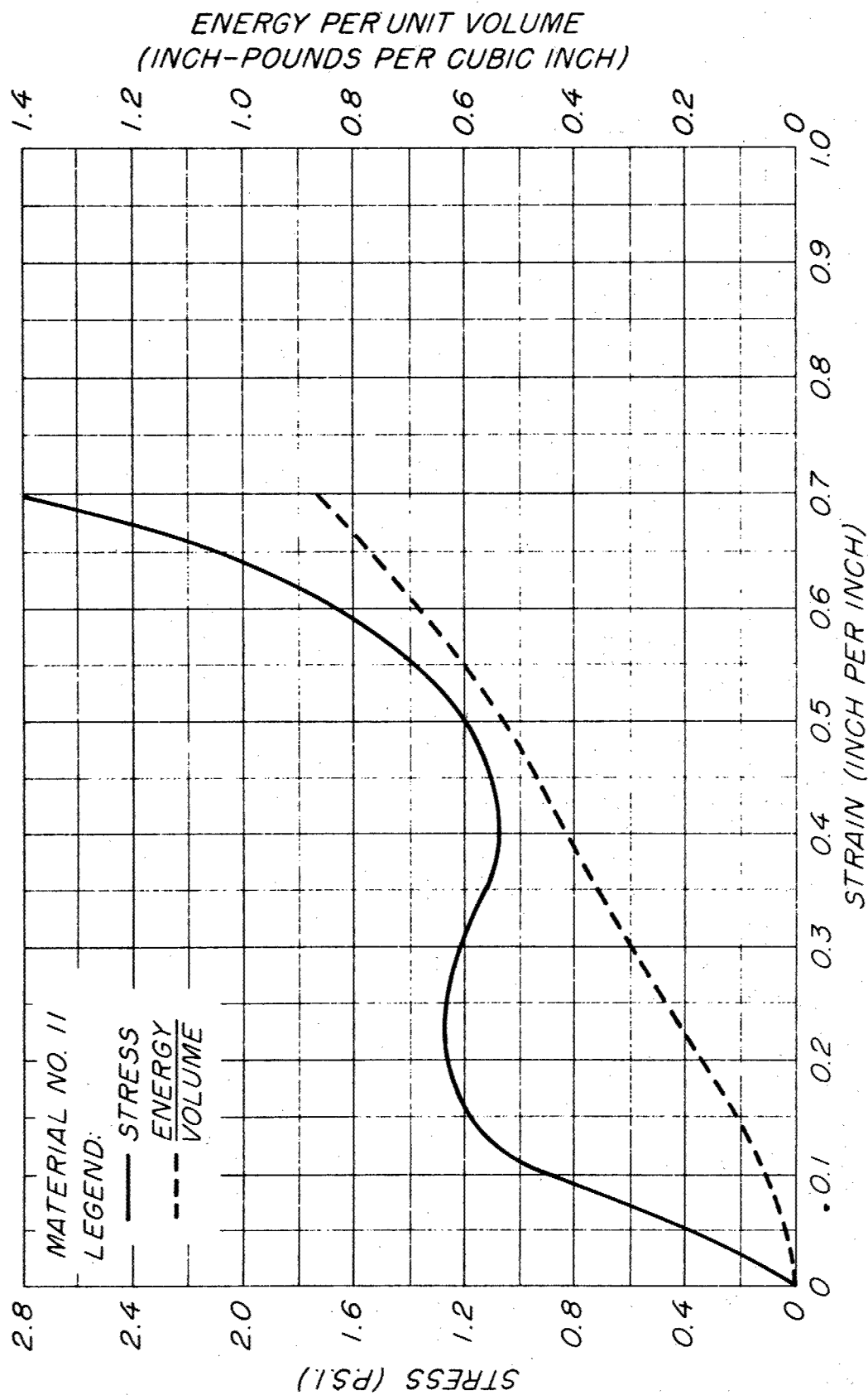


Figure 46.--Stress-strain and energy per unit volume-strain curves for material No. 11
(Stress 0-2.8 P.S.I.).

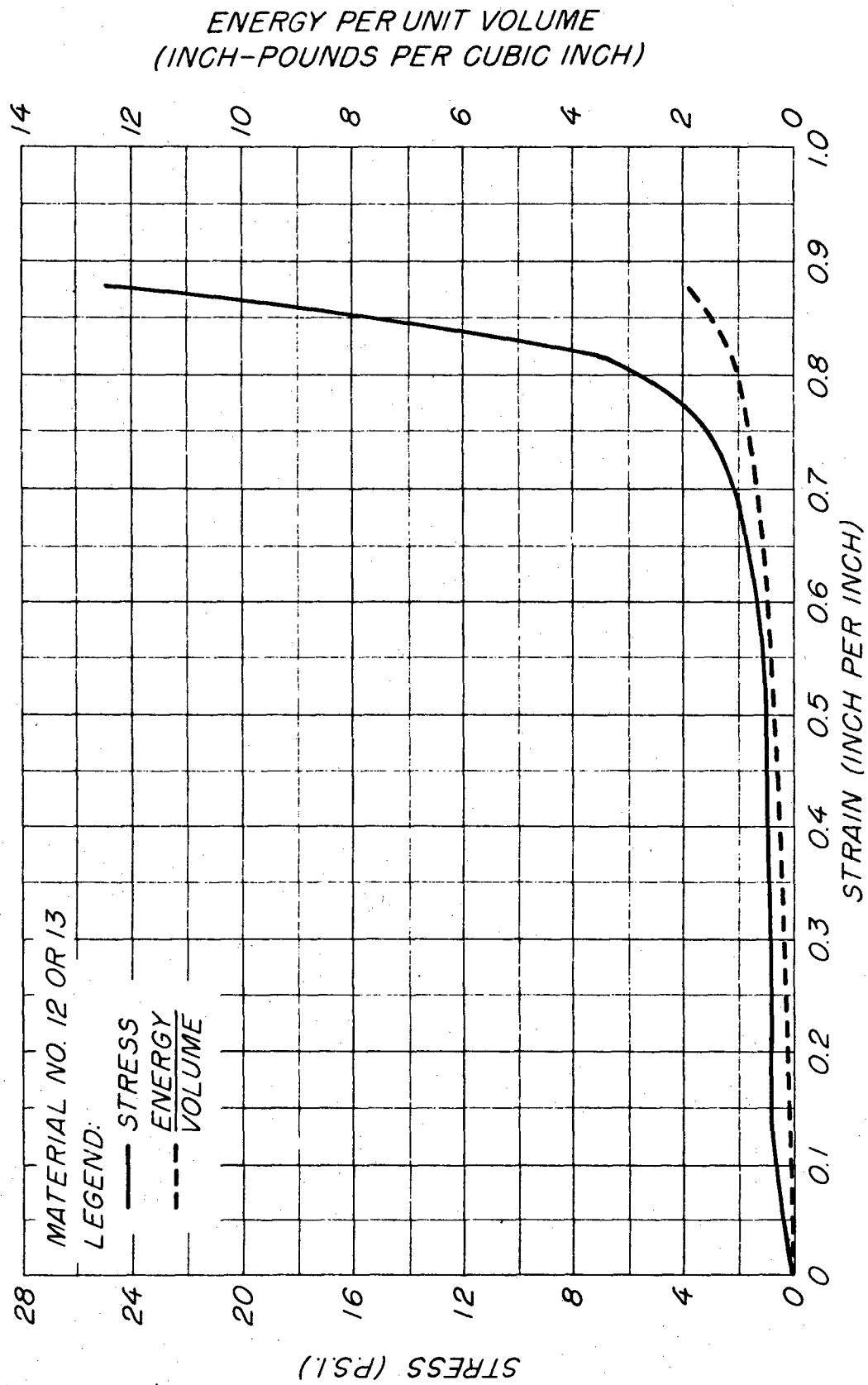


Figure 47. --Stress-strain and energy per unit volume-strain curves for materials Nos. 12 or 13 (Stress 0-28 P.S.I.).

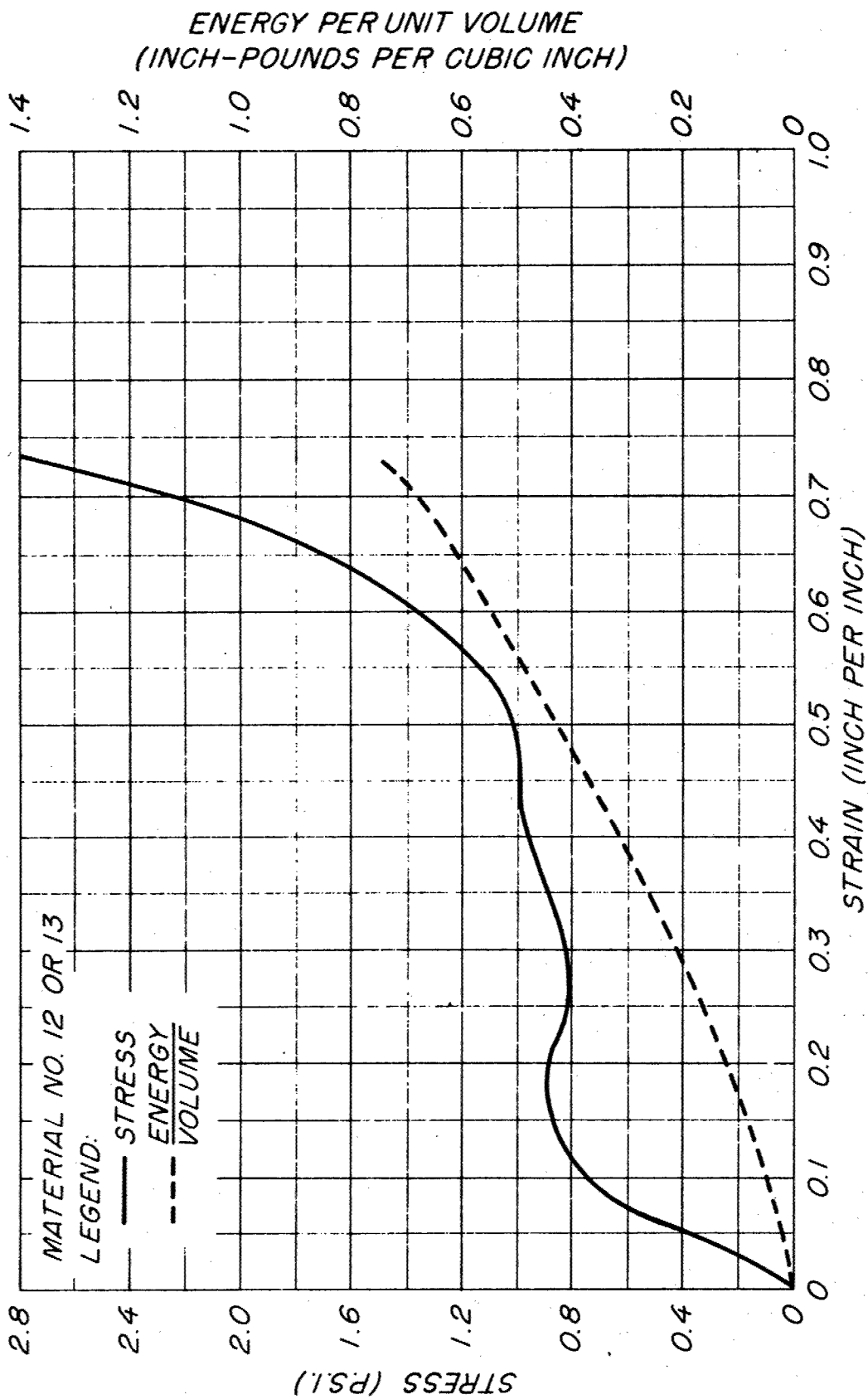


Figure 48.---Stress-strain and energy per unit volume-strain curves for materials Nos. 12 or 13 (Stress 0-2.8 P.S.I.).

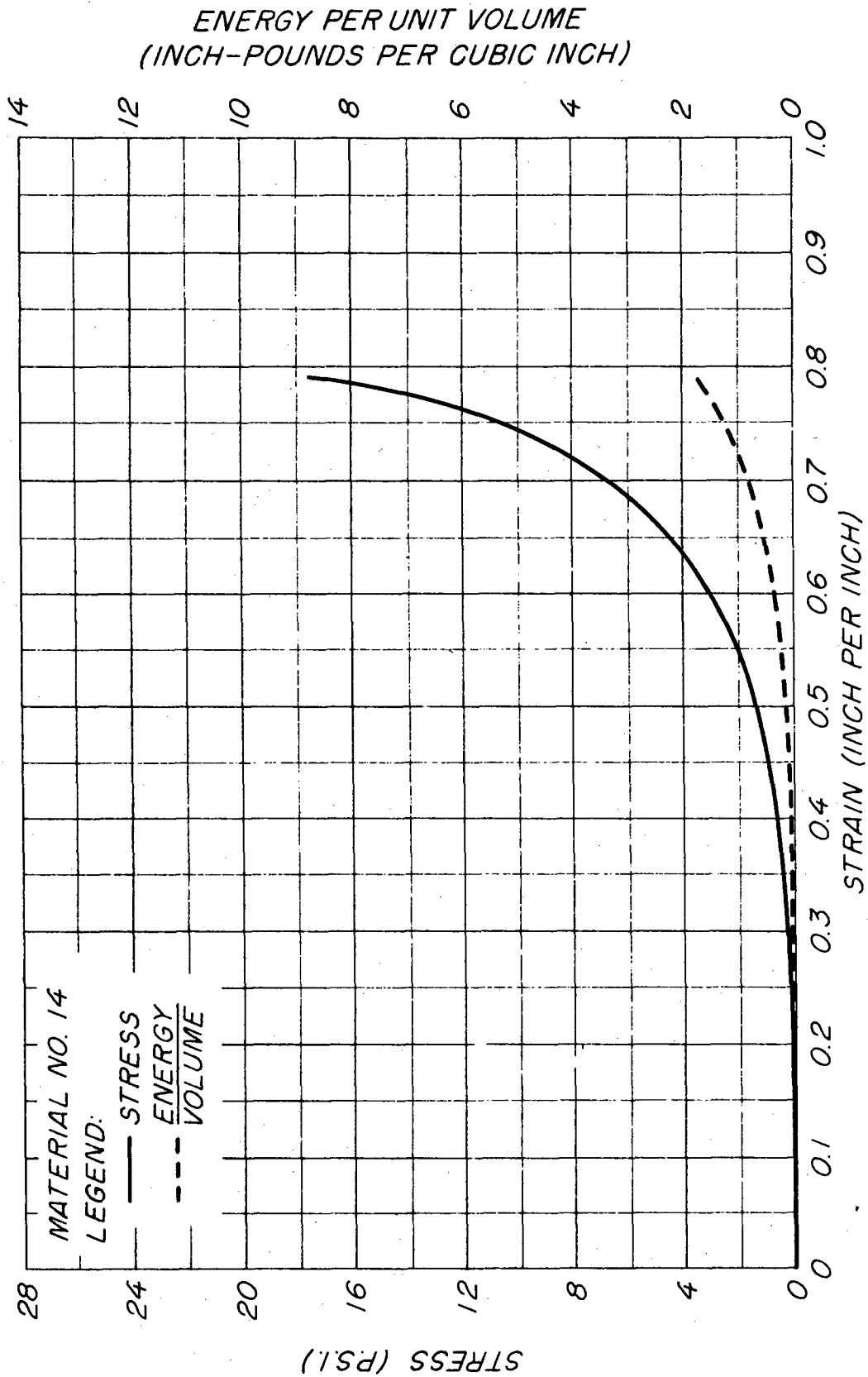


Figure 49.--Stress-strain and energy per unit volume-strain curves for material No. 14
(Stress 0-28 P.S.I.).

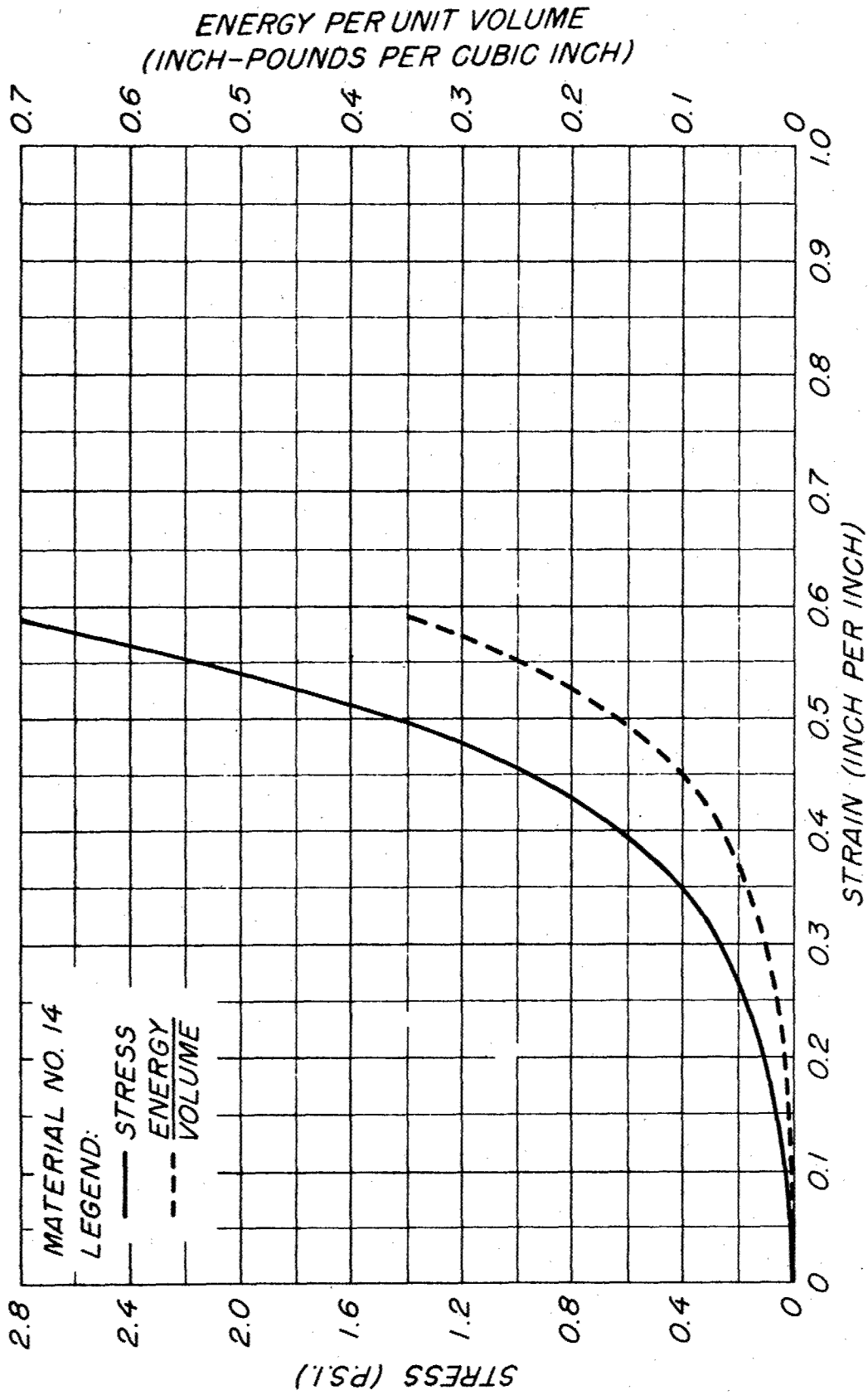


Figure 50.--Stress-strain and energy per unit volume-strain curves for material No. 14
(Stress 0-2.8 P.S.I.).

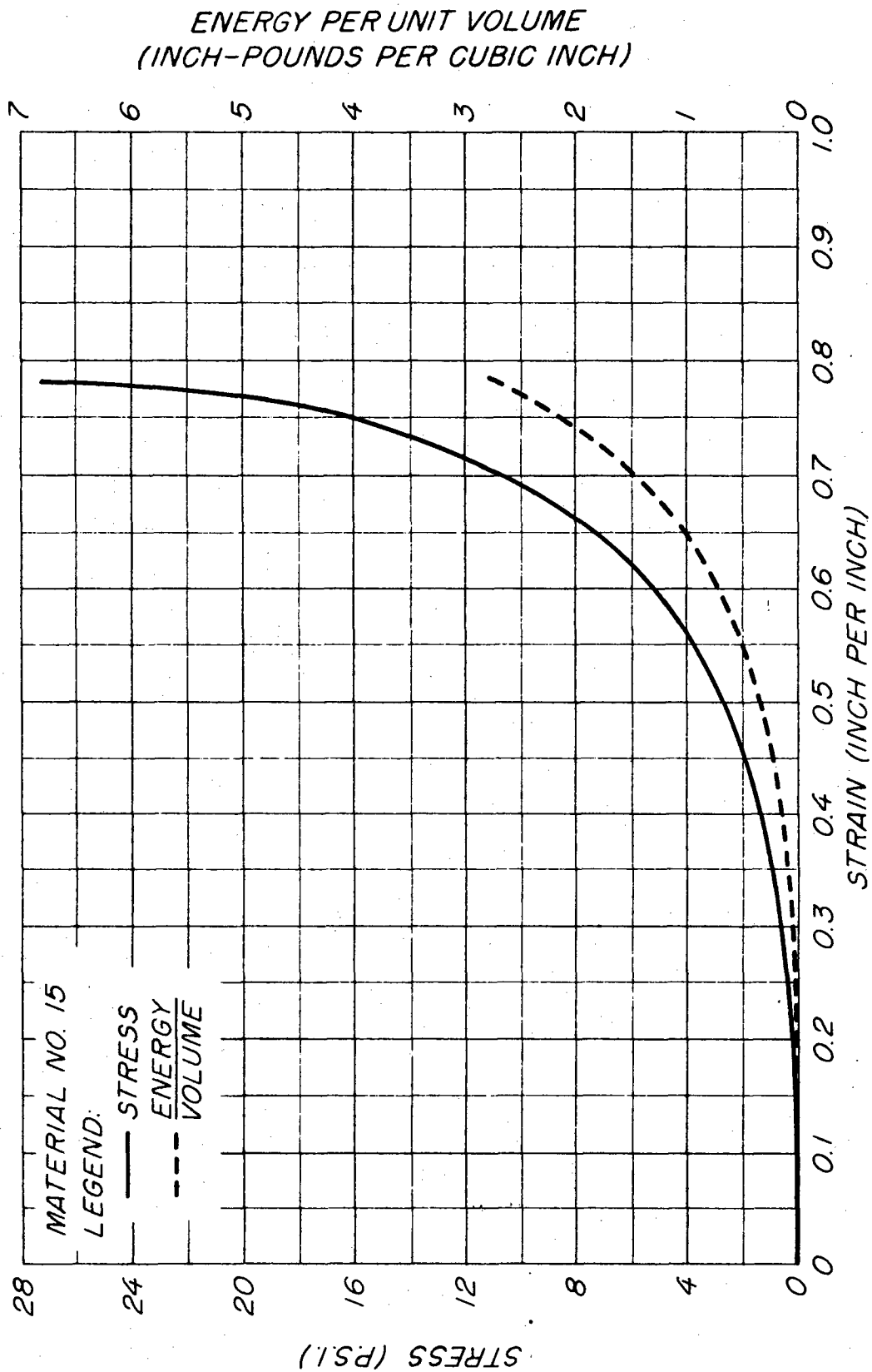


Figure 51.--Stress-strain and energy per unit volume-strain curves for material No. 15
(Stress 0-28 P.S.I.).

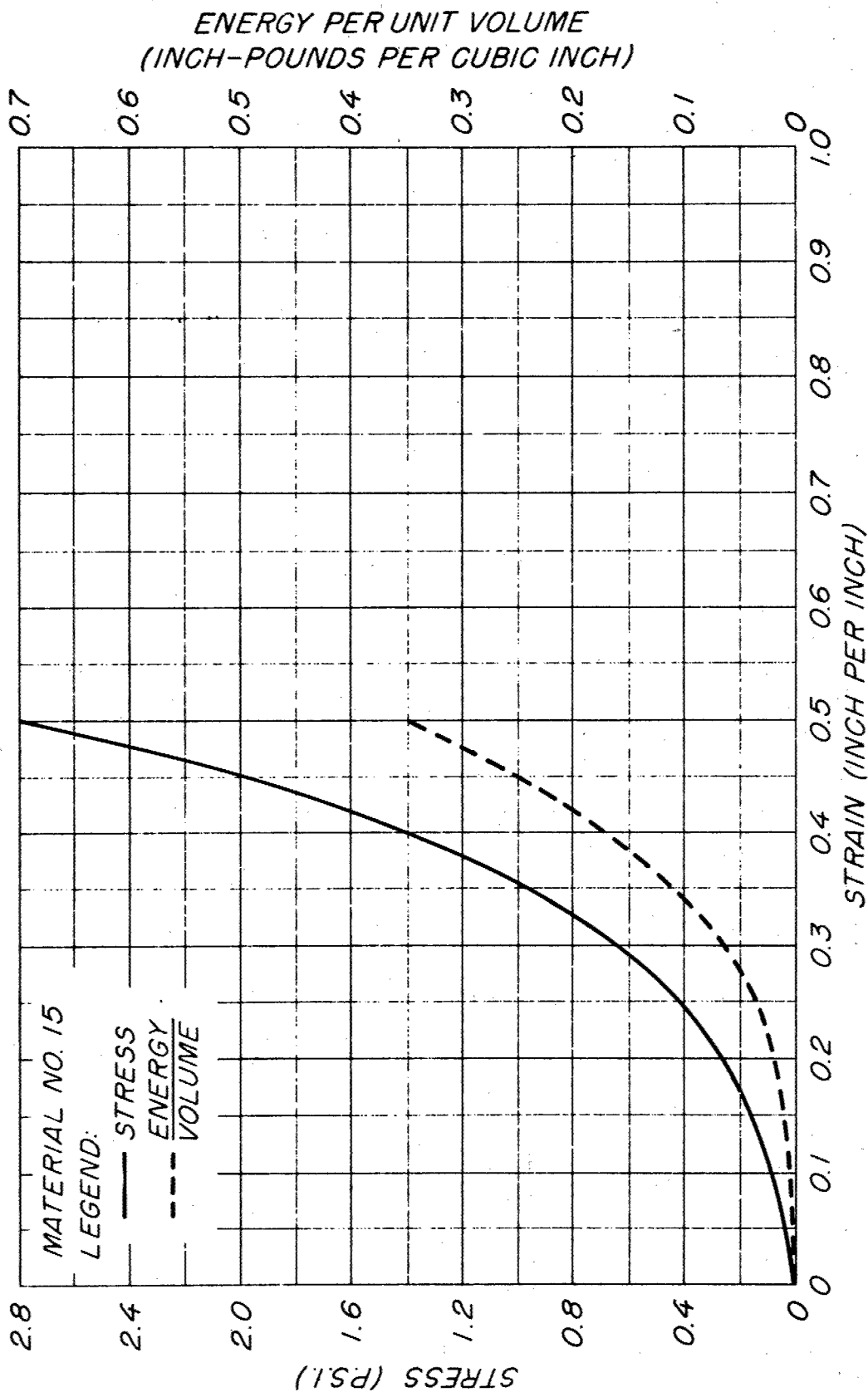


Figure 52.--Stress-strain and energy per unit volume-strain curves for material No. 15
Stress 0-2.8 P.S.I.).

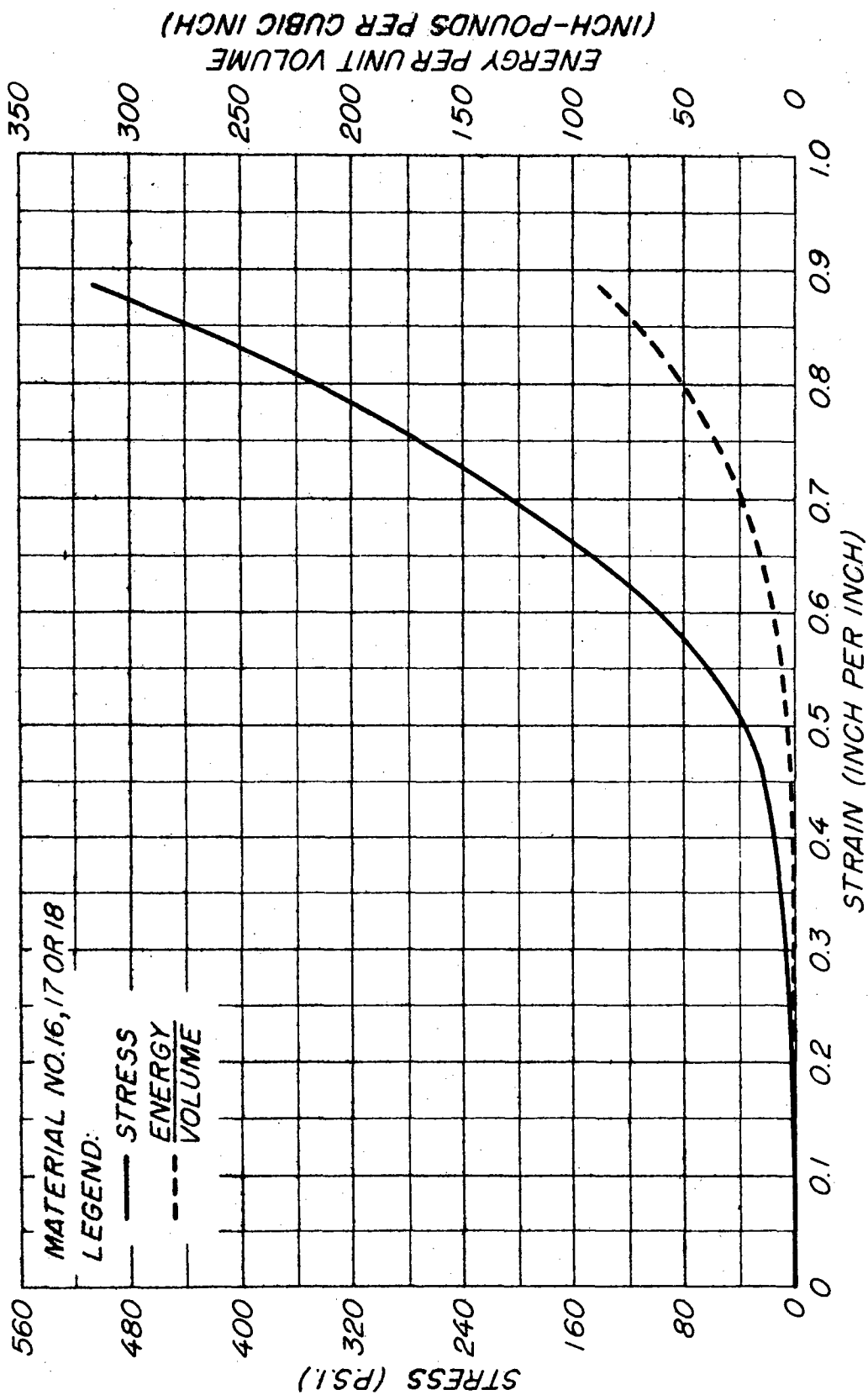


Figure 53.---Stress-strain and energy per unit volume-strain curves for material Nos. 16, 17, or 18 (Stress 0-560 P.S.I.).

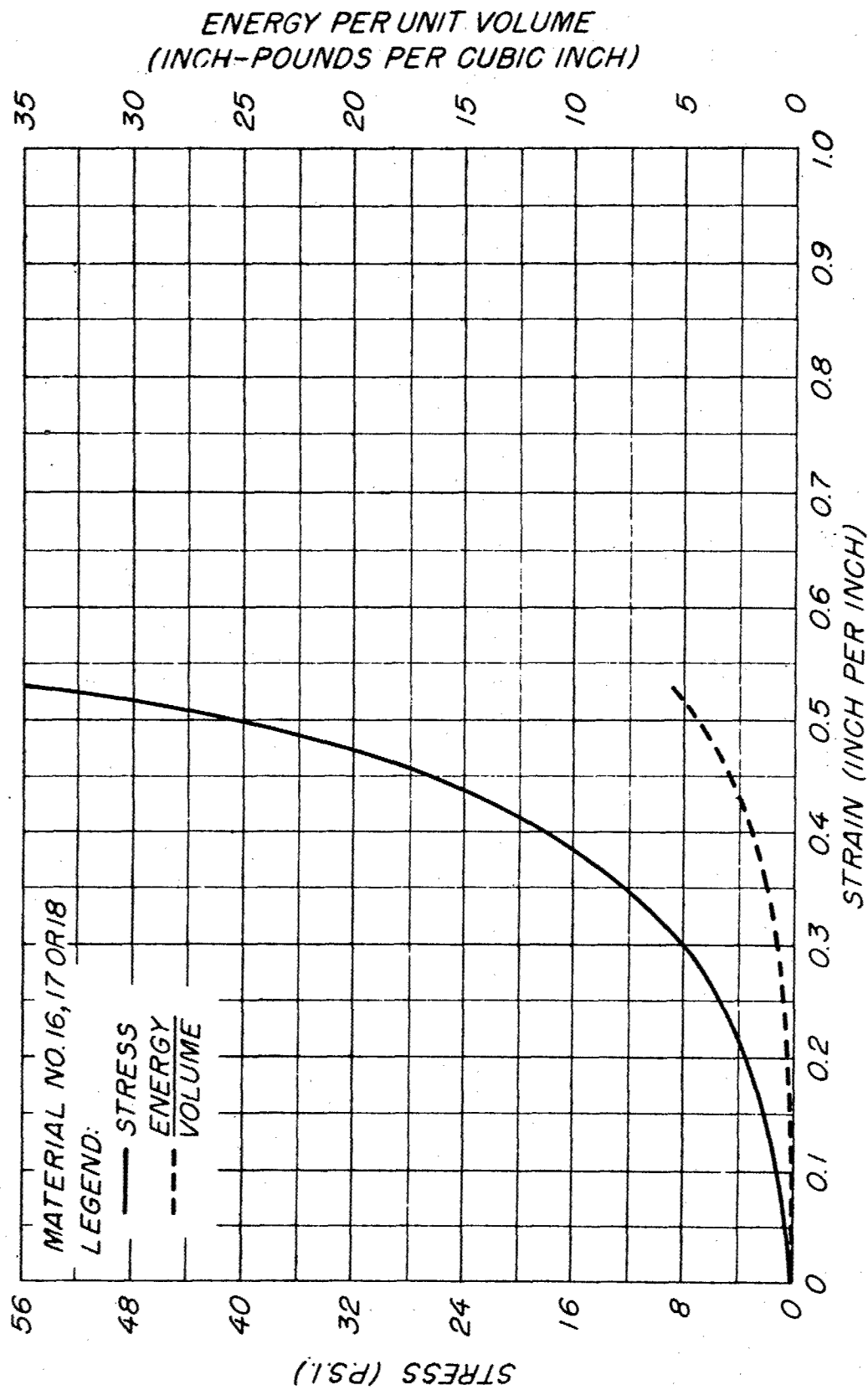


Figure 54.--Stress-strain and energy per unit volume-strain curves for material Nos. 16, 17, or 18 (Stress 0-56 P.S.I.).

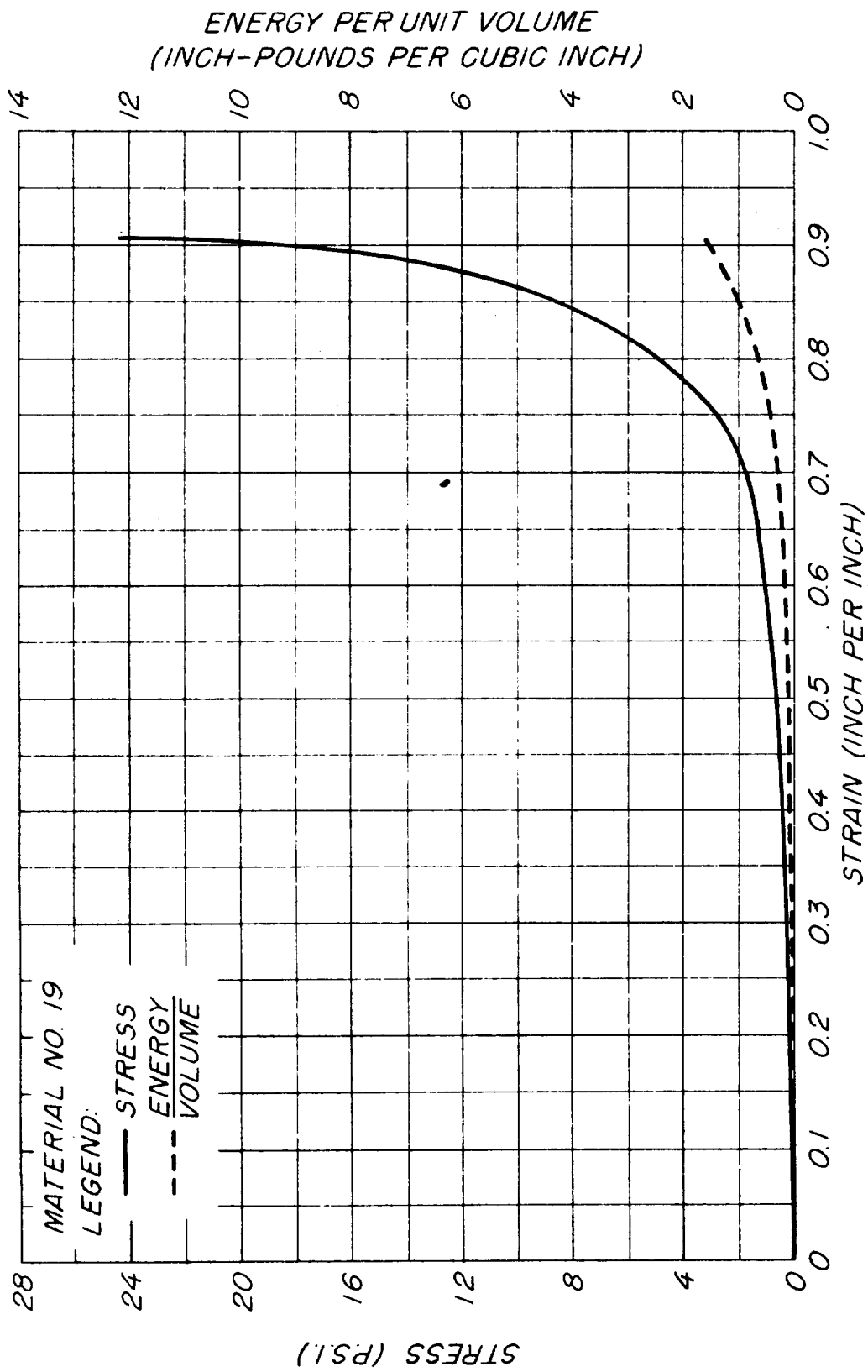


Figure 55. --Stress-strain and energy per unit volume-strain curves for material No. 19
(Stress 0-28 P.S.I.).

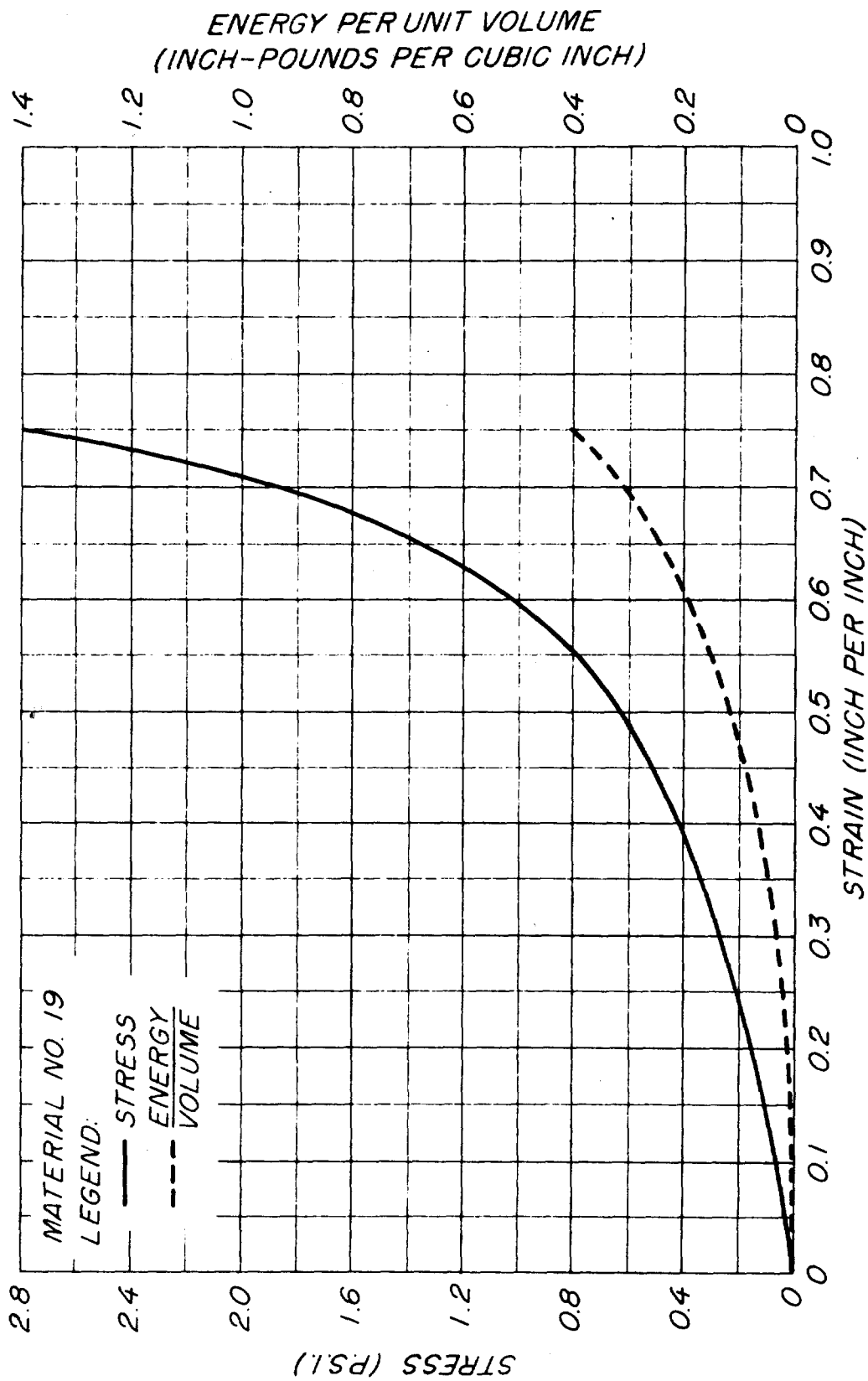


Figure 56.---Stress-strain and energy per unit volume-strain curves for material No. 19
(Stress 0-2.8 P.S.I.).

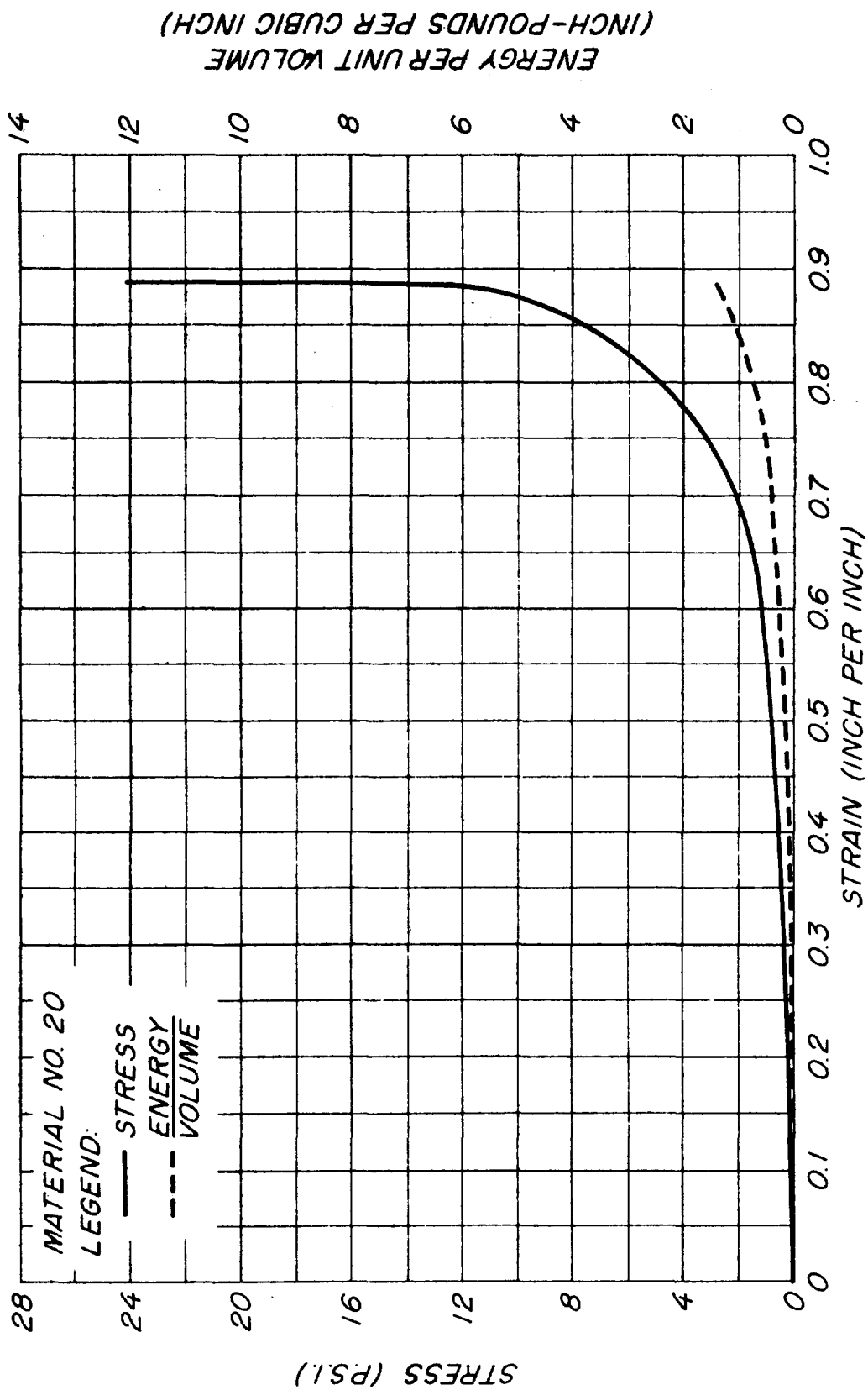


Figure 57.---Stress-strain and energy per unit volume-strain curves for material No. 20
(Stress 0-28 P.S.I.).

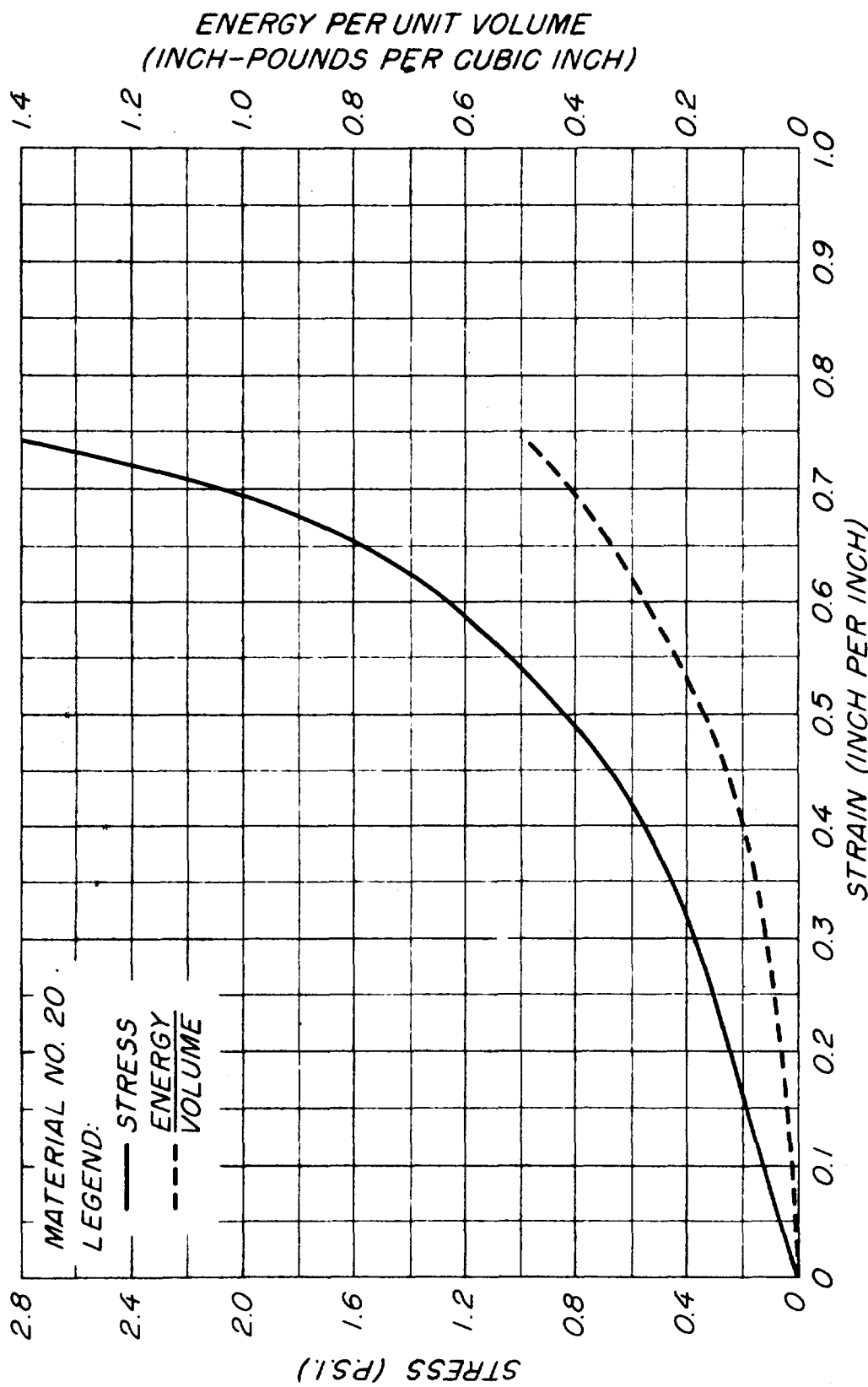


Figure 58. --- Stress-strain and energy per unit volume-strain curves for material No. 20
(Stress 0-2.8 P.S.I.).

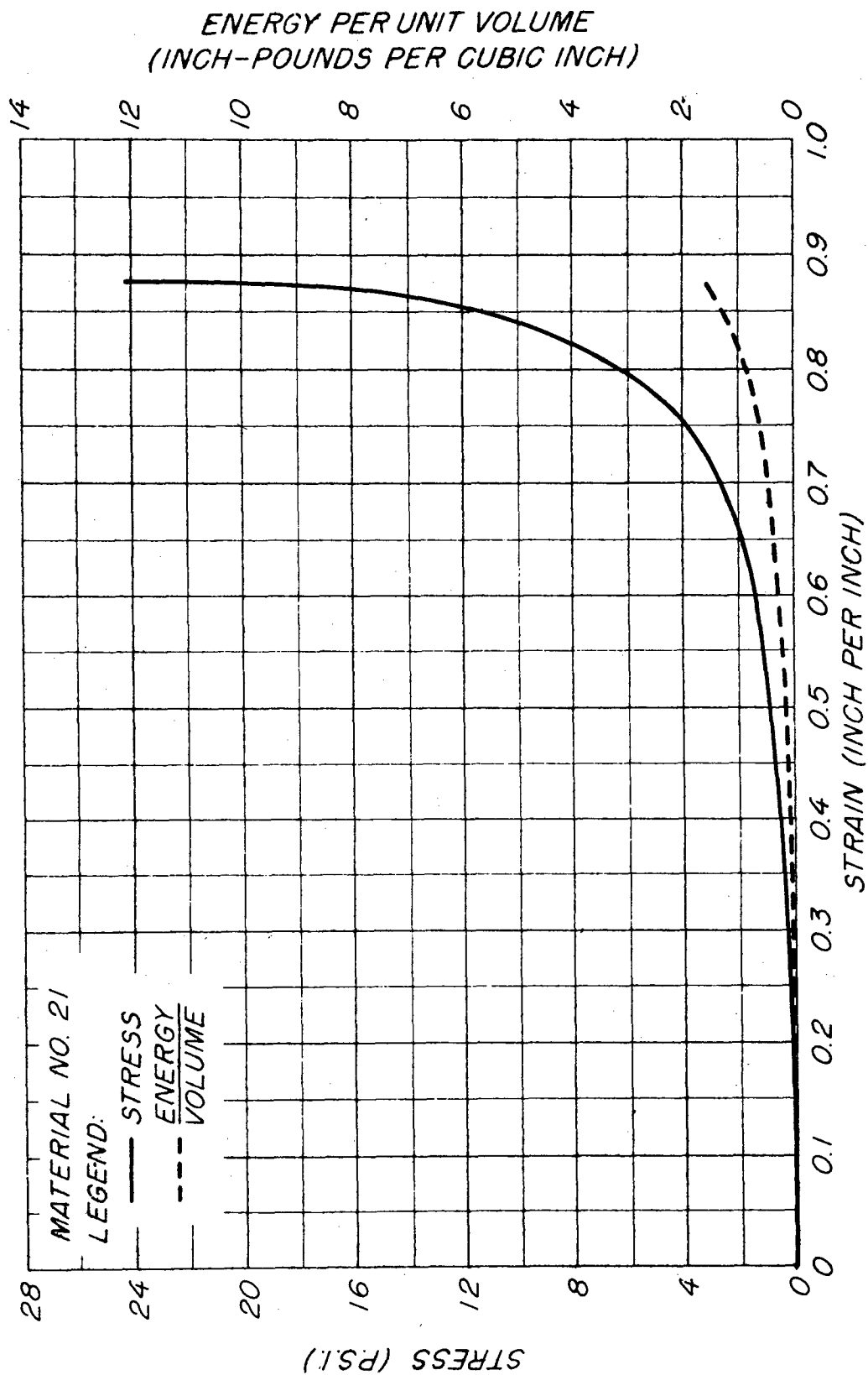


Figure 59. --Stress-strain and energy per unit volume-strain curves for material No. 21
(Stress 0-28 P.S.I.).

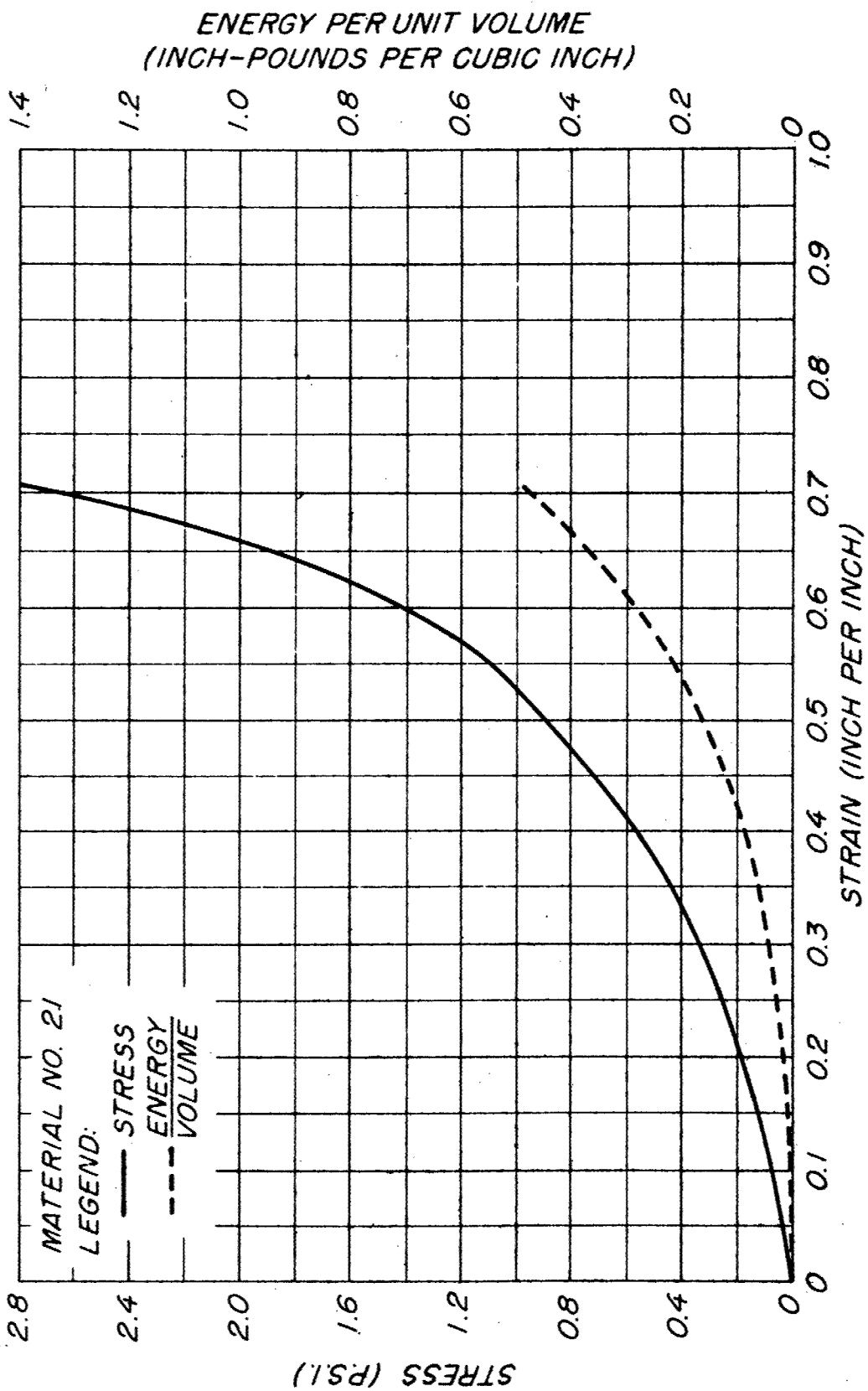


Figure 60.--Stress-strain and energy per unit volume-strain curves for material No. 21
(Stress 0-2.8 P.S.I.).

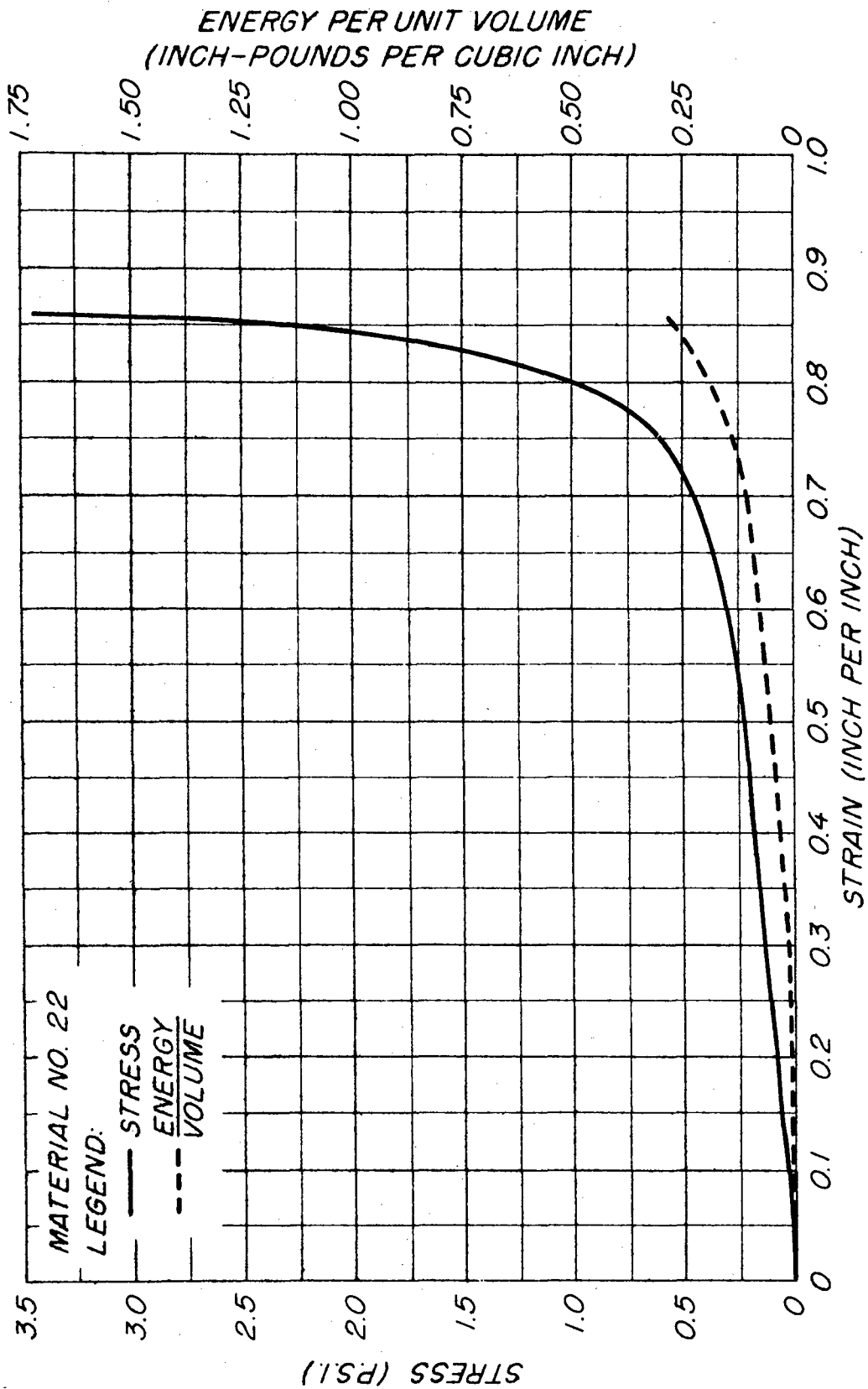


Figure 61.--Stress-strain and energy per unit volume-strain curves for material No. 22
(Stress 0-3.5 P.S.I.).

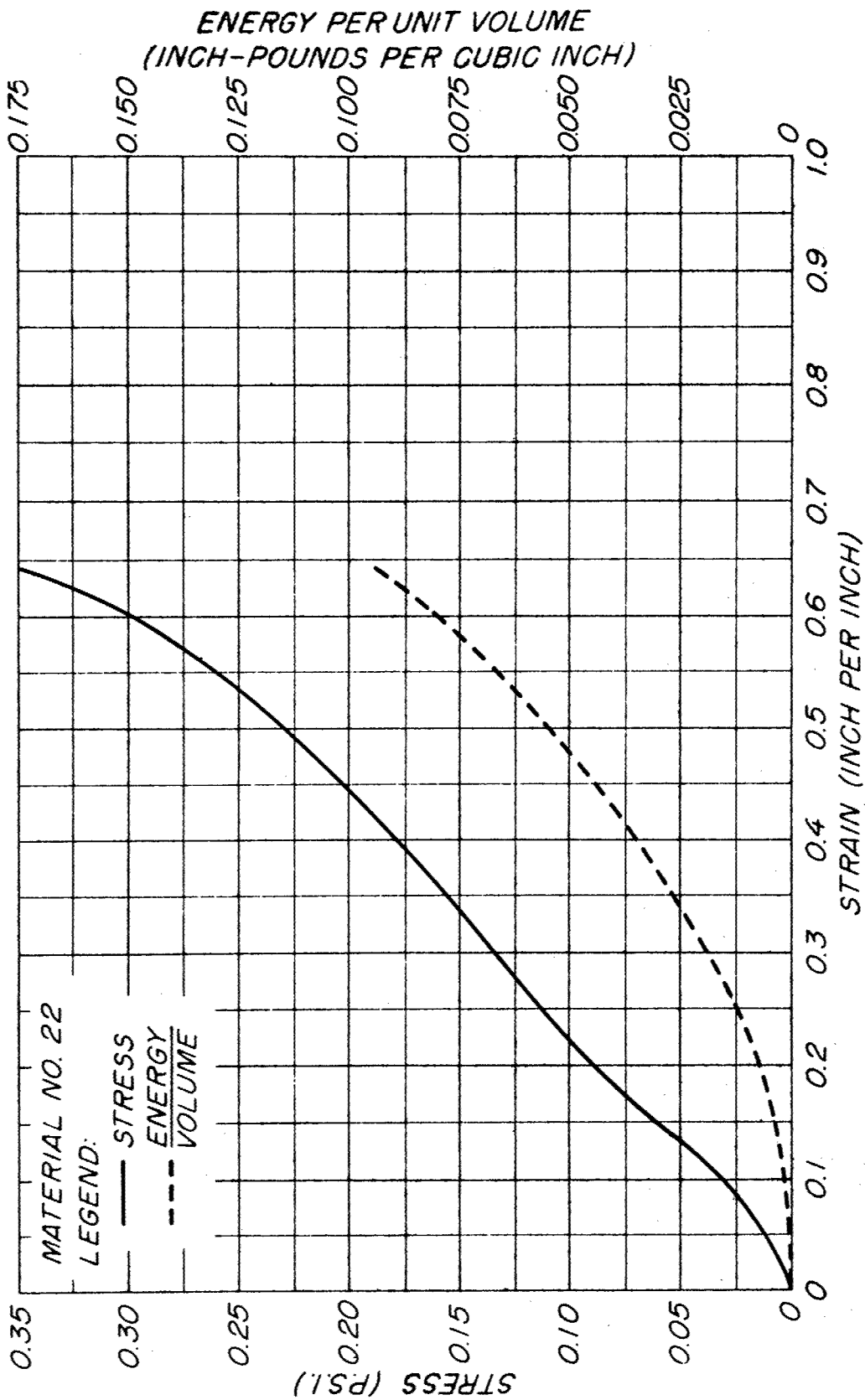


Figure 62.--Stress-strain and energy per unit volume-strain curves for material No. 22
(Stress 0-0.35 P.S.I.).

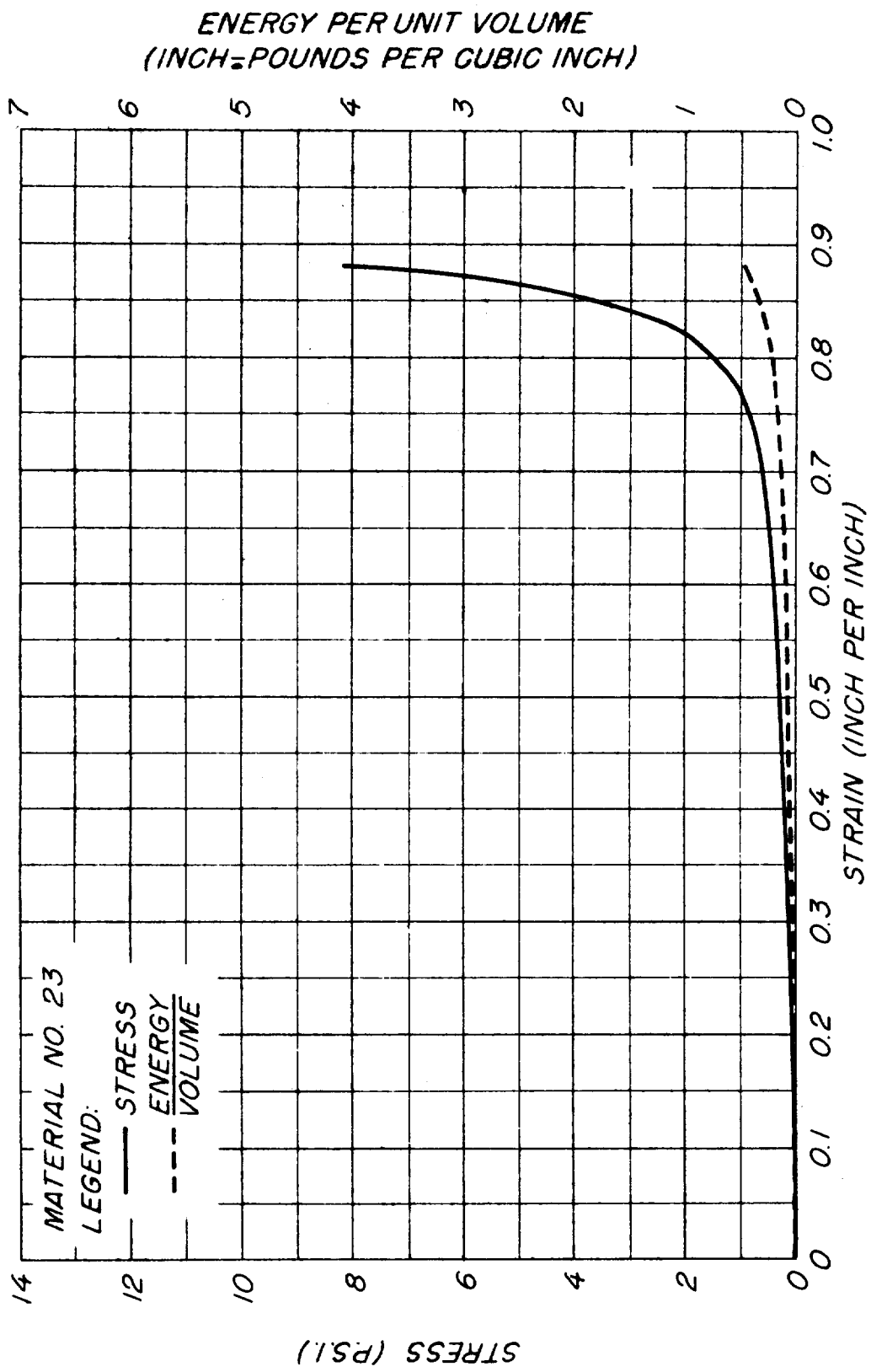


Figure 63.---Stress-strain and energy per unit volume-strain curves for material No. 23
(Stress 0-14 P.S.I.).

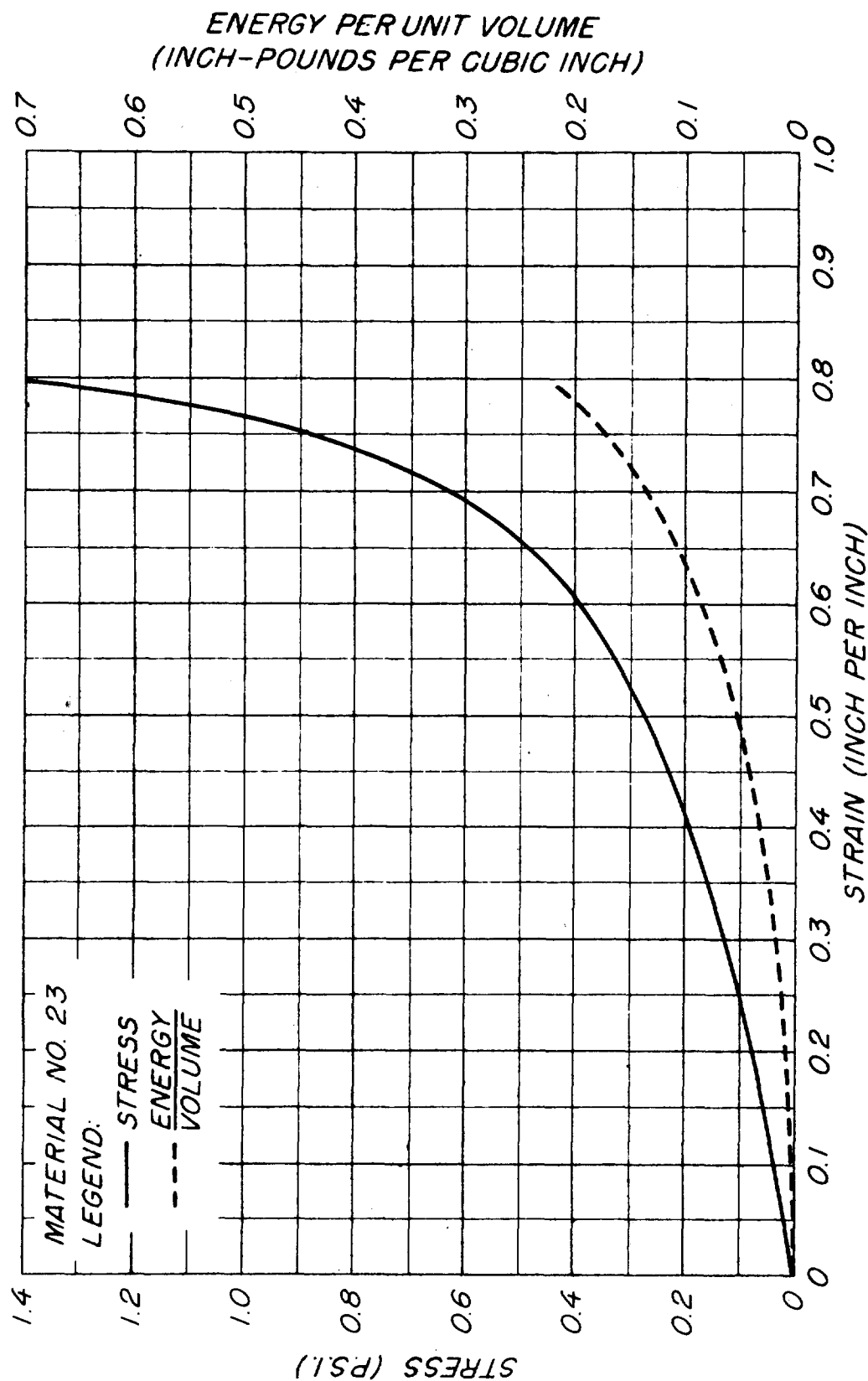


Figure 64.---Stress-strain and energy per unit volume-strain curves for material No. 23
(Stress 0-1.4 P.S.I.).

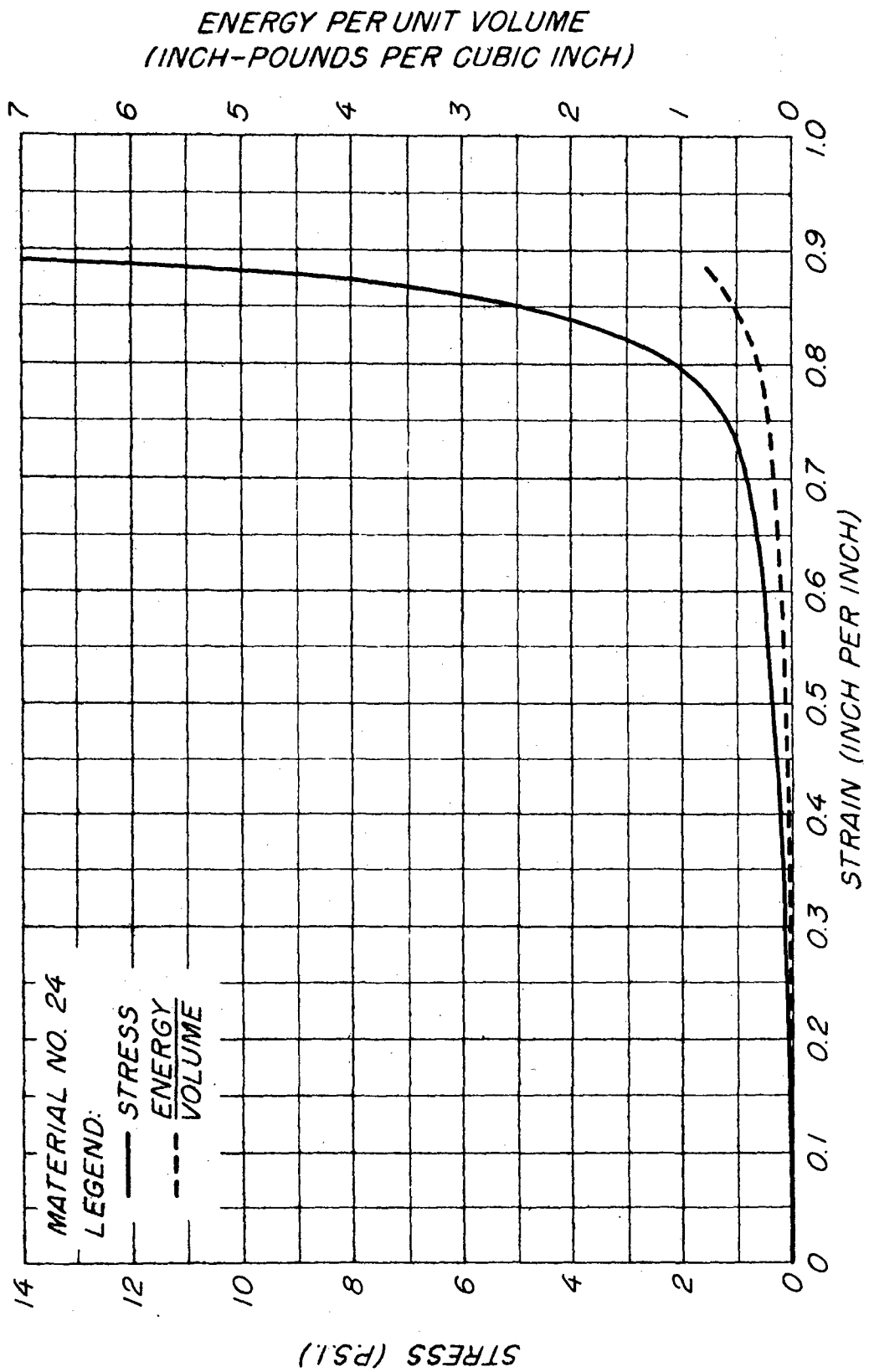


Figure 65.---Stress-strain and energy per unit volume-strain curves for material No. 24
(Stress 0-14 P.S.I.).

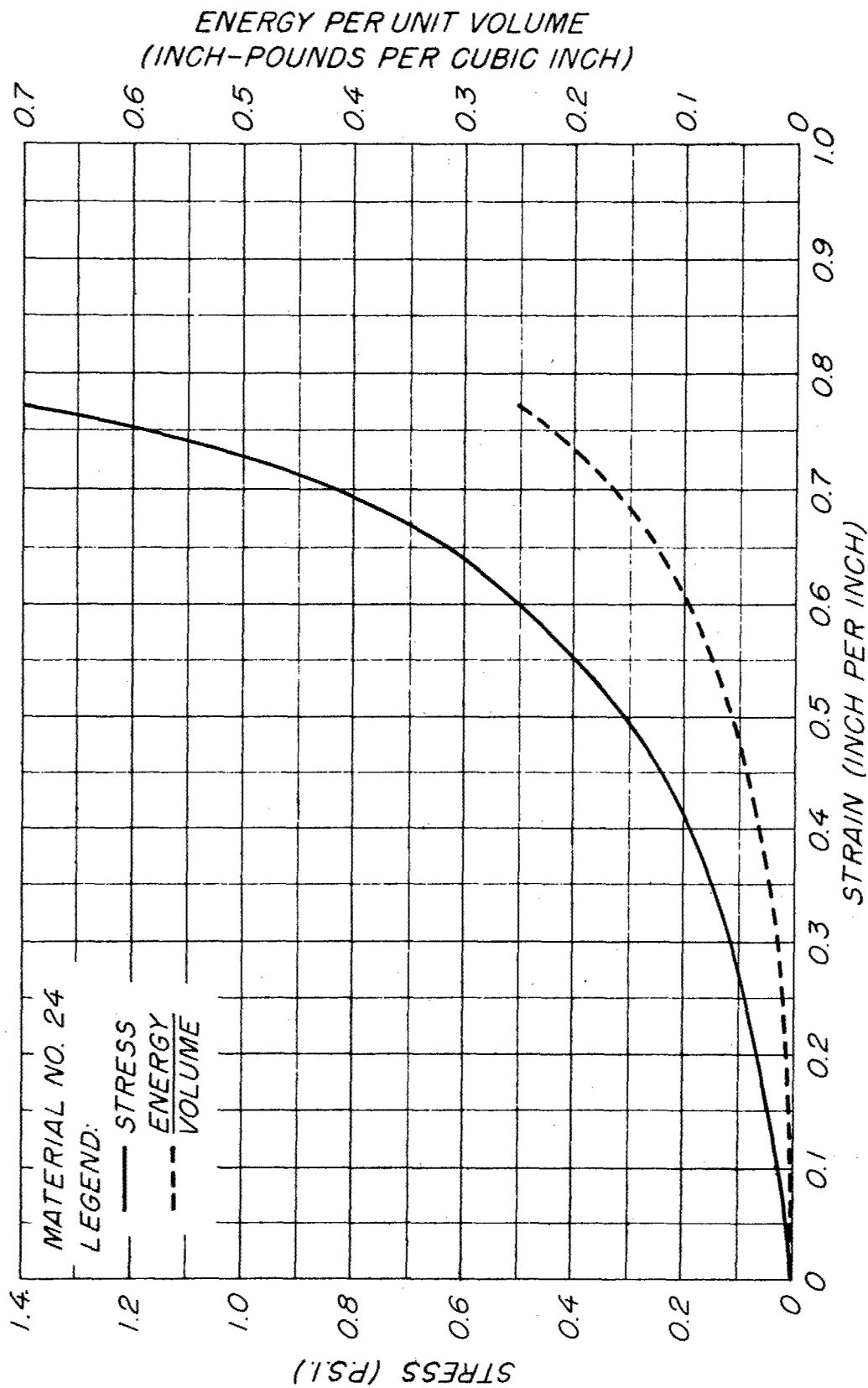


Figure 66.--Stress-strain and energy per unit volume-strain curves for material No. 24
(Stress 0-1.4 P.S.I.).

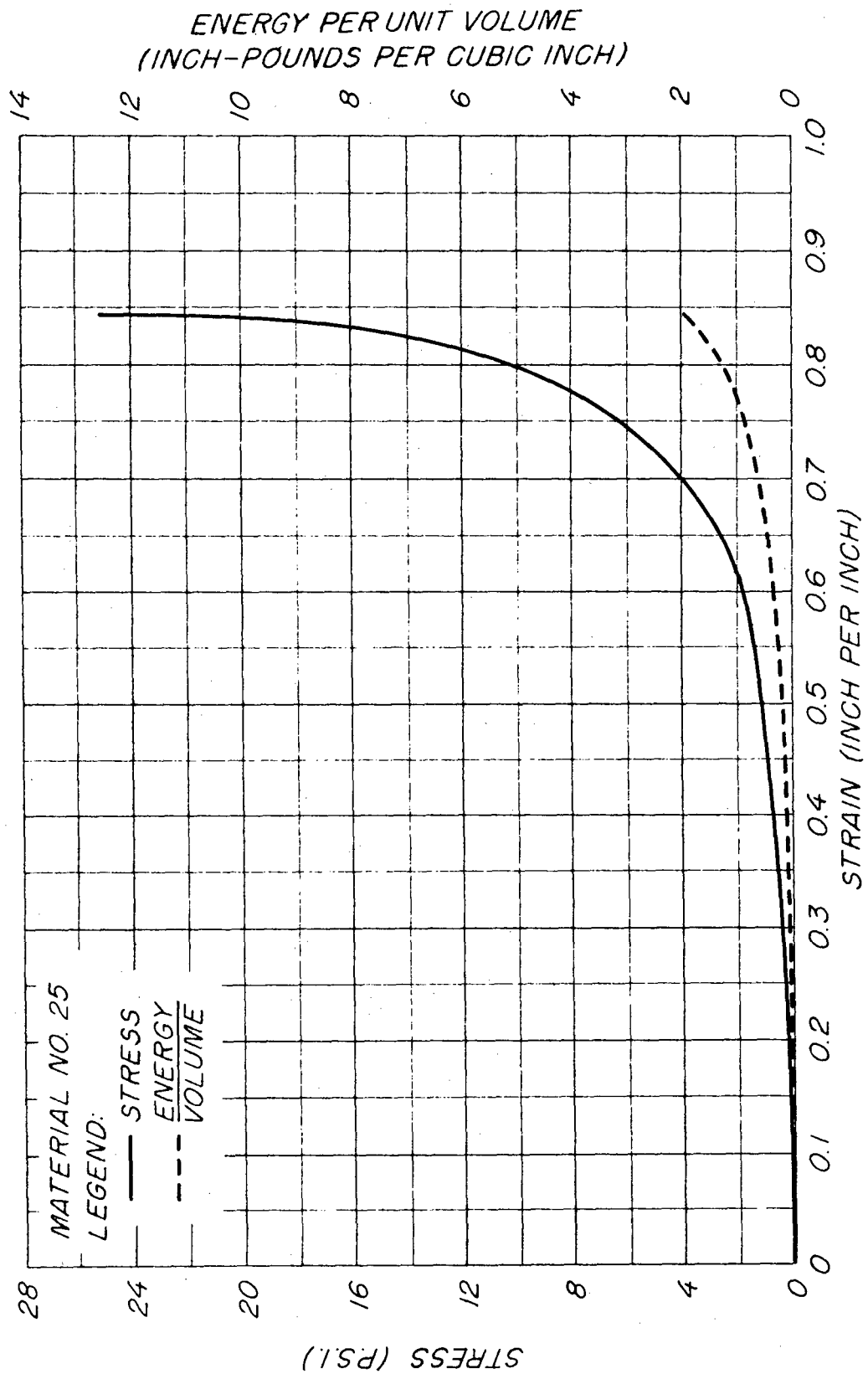


Figure 67.---Stress-strain and energy per unit volume-strain curves for material No. 25
 (Stress 0-28 P.S.I.).

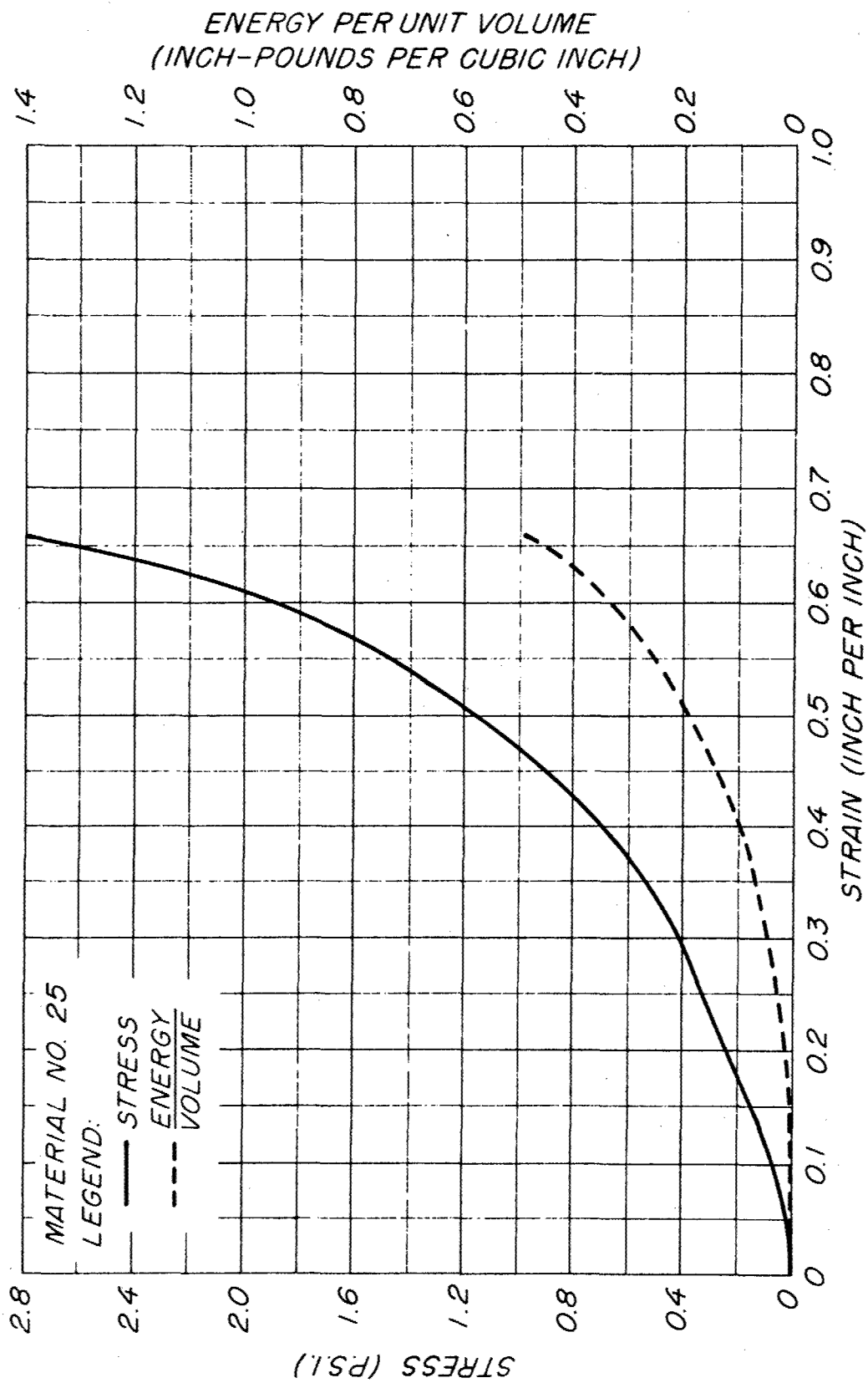


Figure 68.---Stress-strain and energy per unit volume-strain curves for material No. 25
(Stress 0-2.8 P.S.I.).

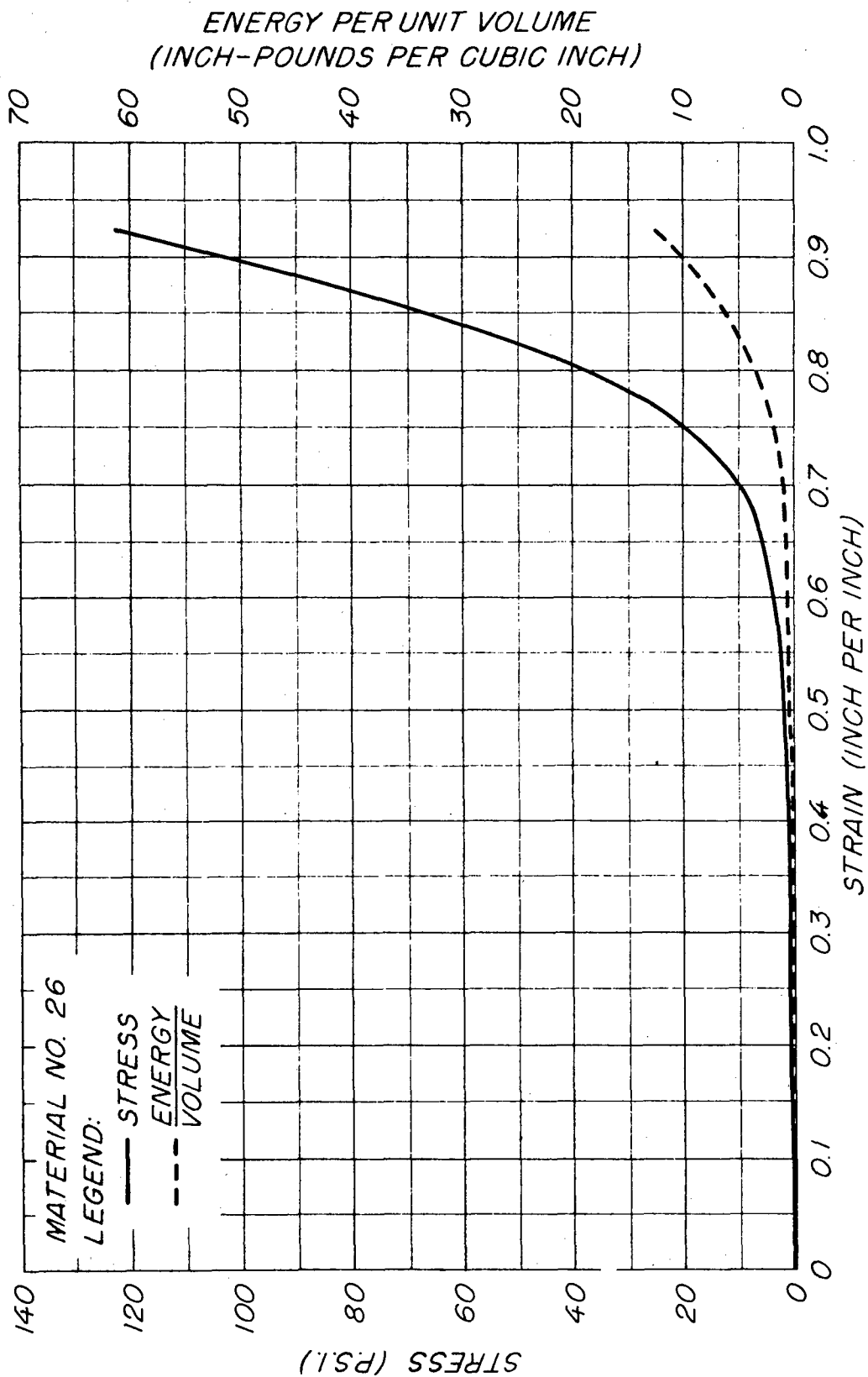


Figure 69.---Stress-strain and energy per unit volume-strain curves for material No. 26
(Stress 0-140 P.S.I.).

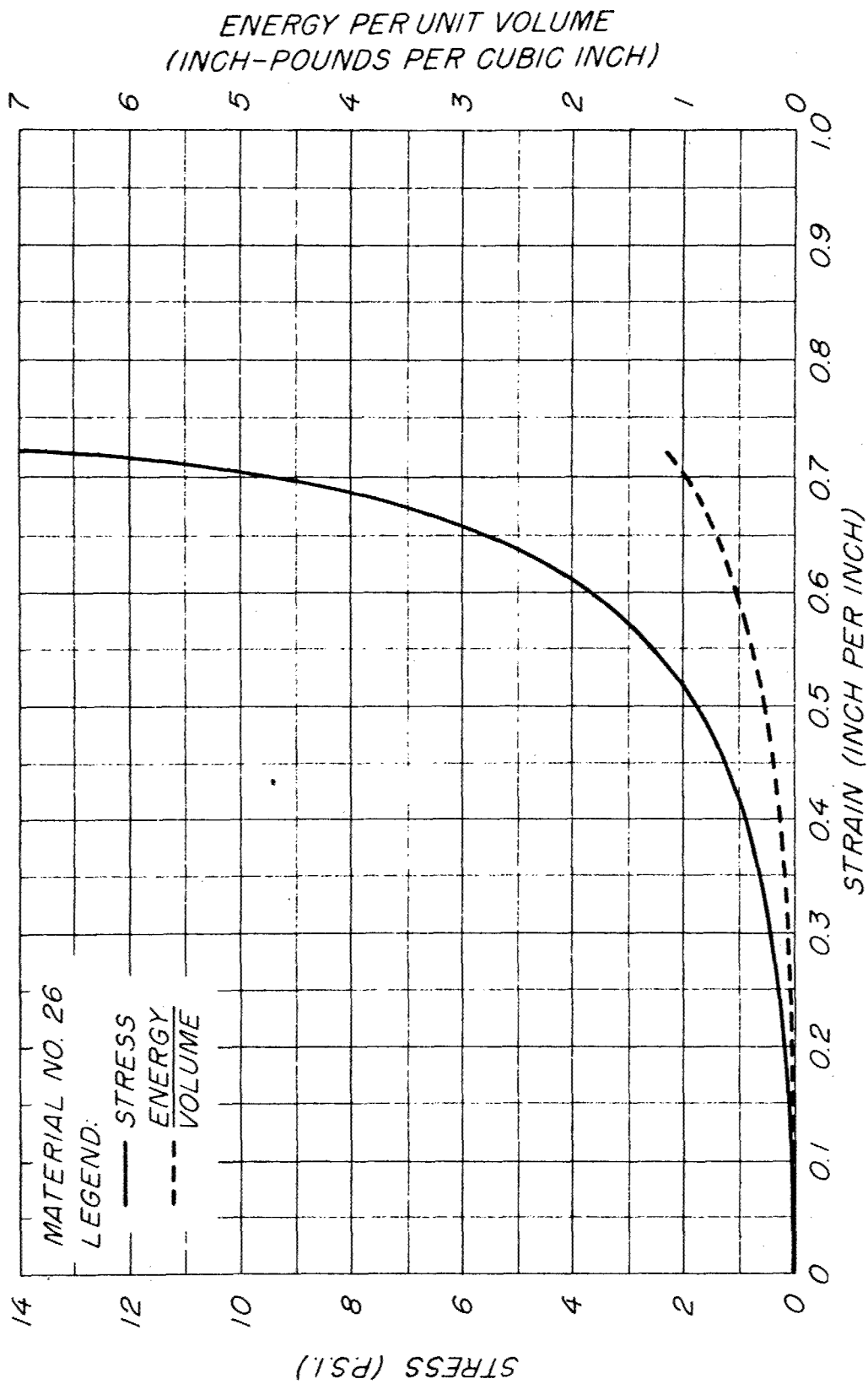


Figure 70.---Stress-strain and energy per unit volume-strain curves for material No. 26
(Stress 0-14 P.S.I.).

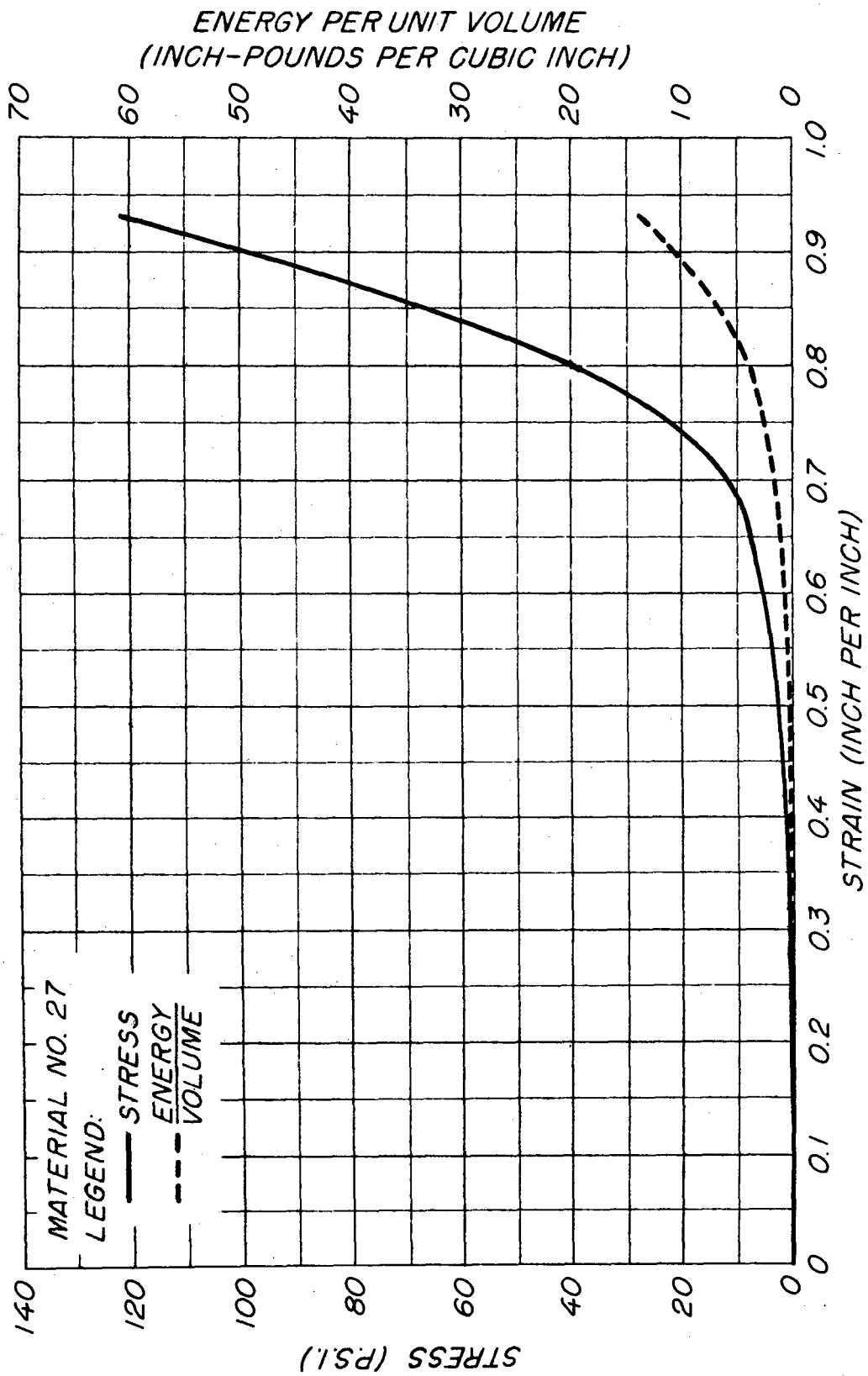


Figure 71.--Stress-strain and energy per unit volume-strain curves for material No. 27
(Stress 0-140 P.S.I.).

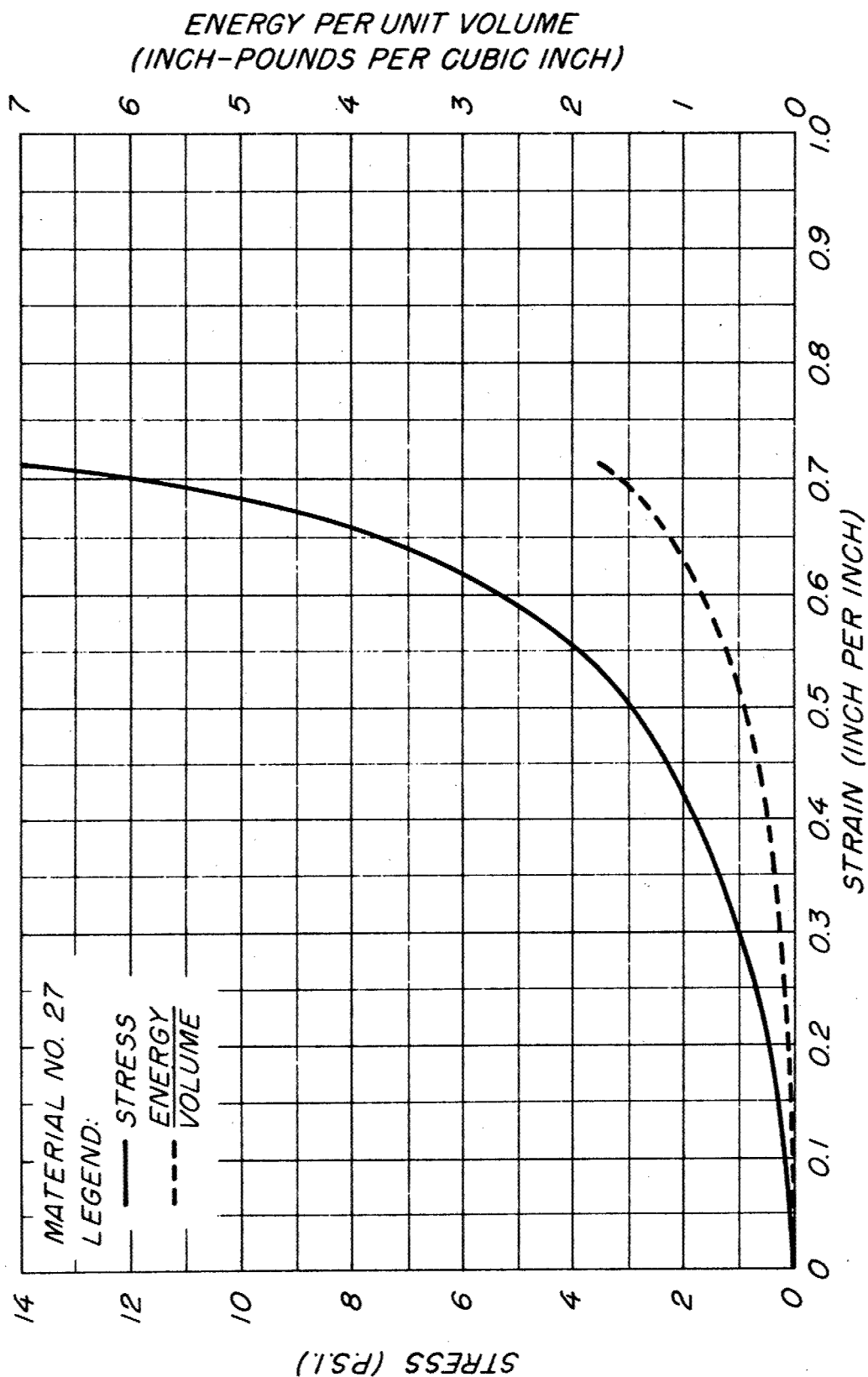


Figure 72.---Stress-strain and energy per unit volume-strain curves for material No. 27
(Stress 0-14 P.S.I.).

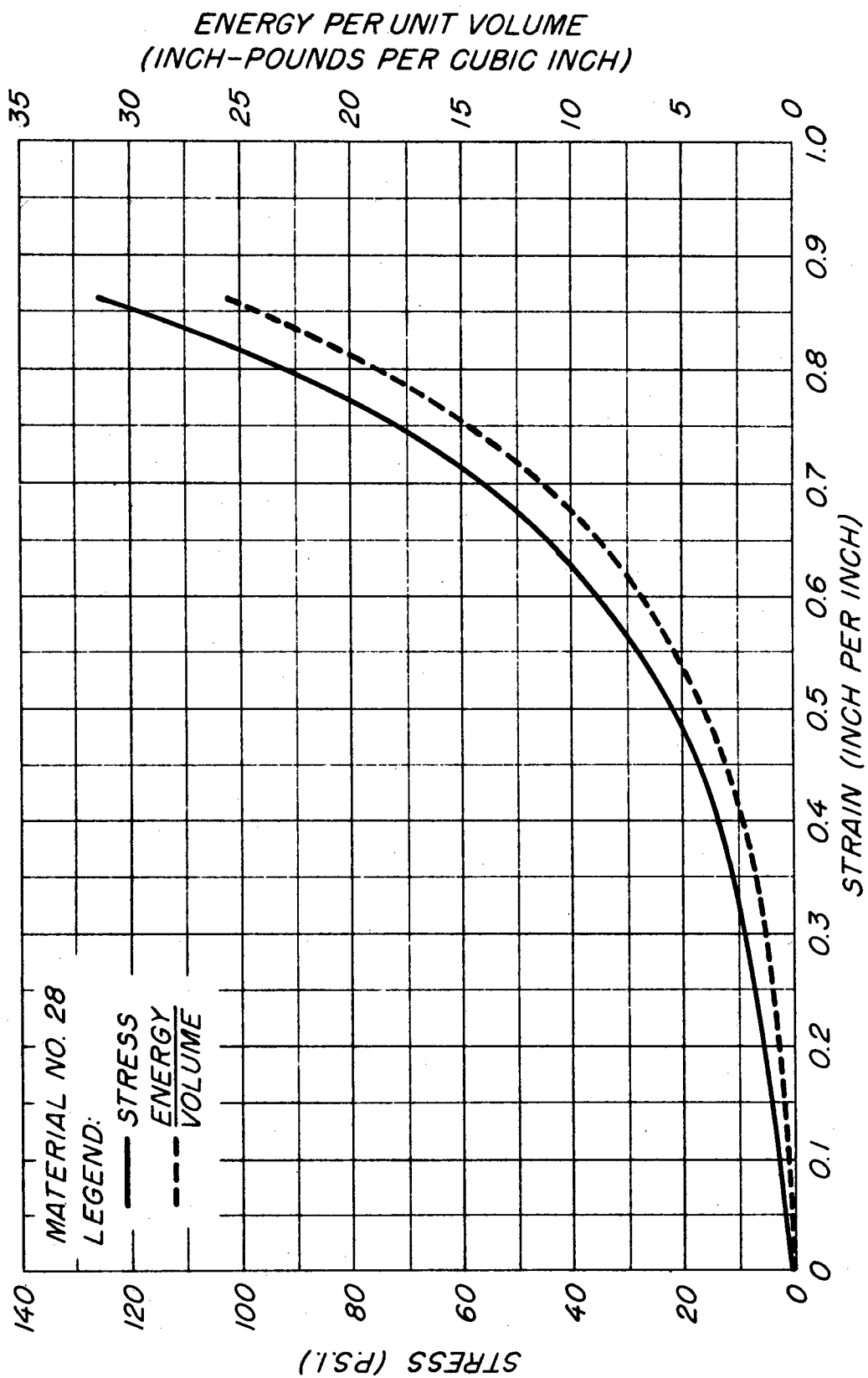


Figure 73.--Stress-strain and energy per unit volume-strain curves for material No. 28
(Stress 0-140 P.S.I.).

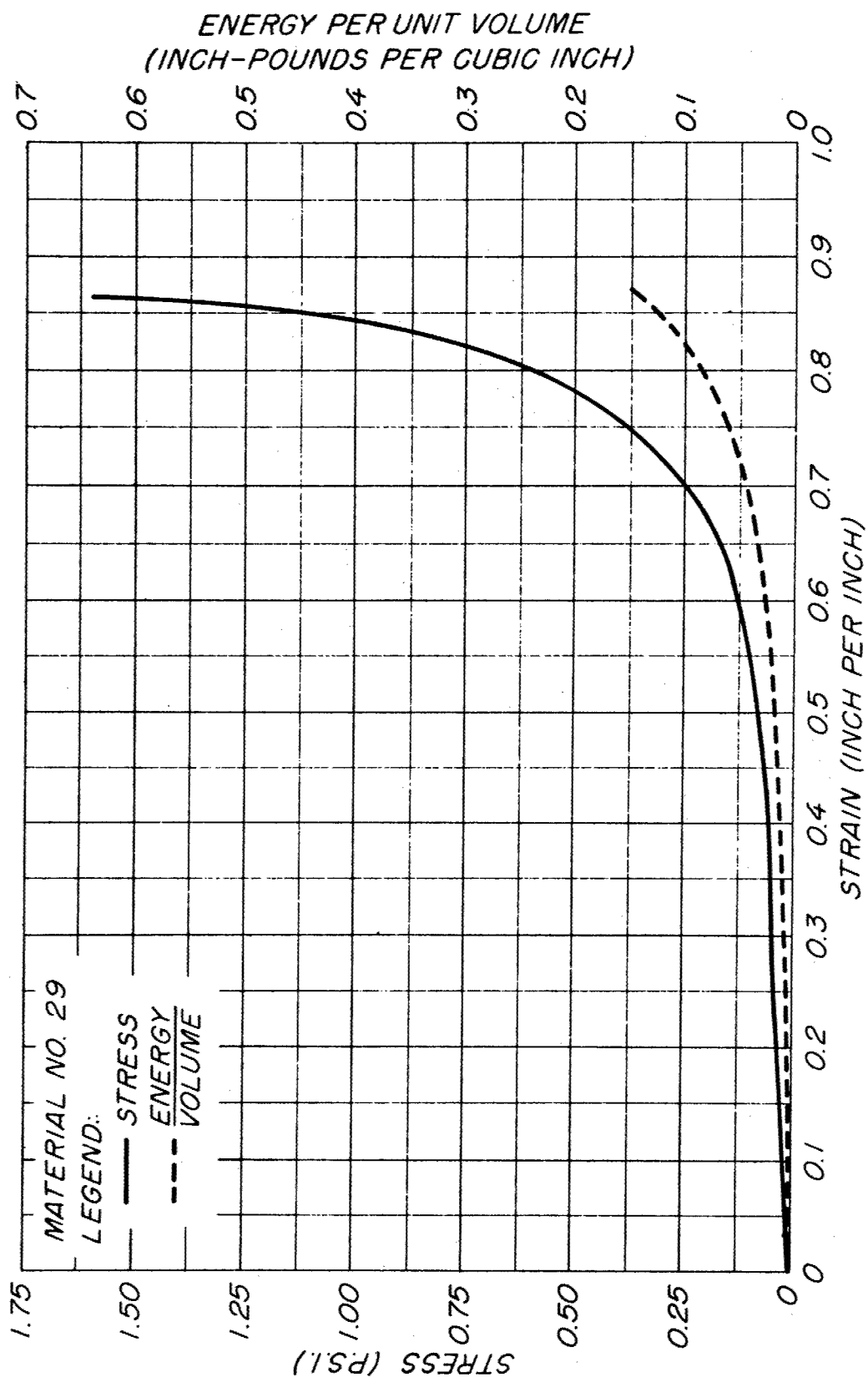


Figure 74.--Stress-strain and energy per unit volume-strain curves for material No. 29
(Stress 0-1.75 P.S.I.).

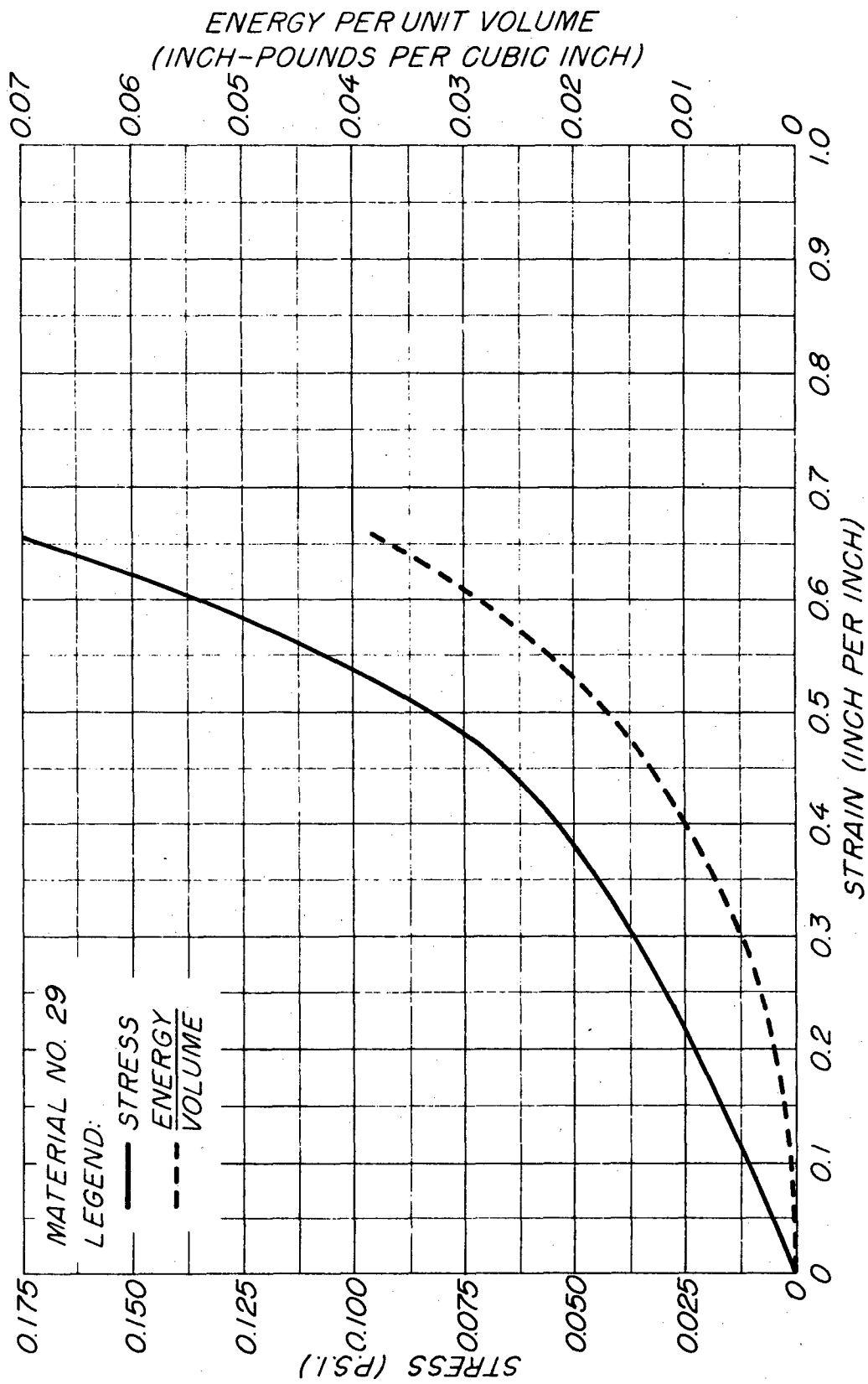


Figure 75. --- Stress-strain and energy per unit volume-strain curves for material No. 29
(Stress 0-0.175 P.S.I.).

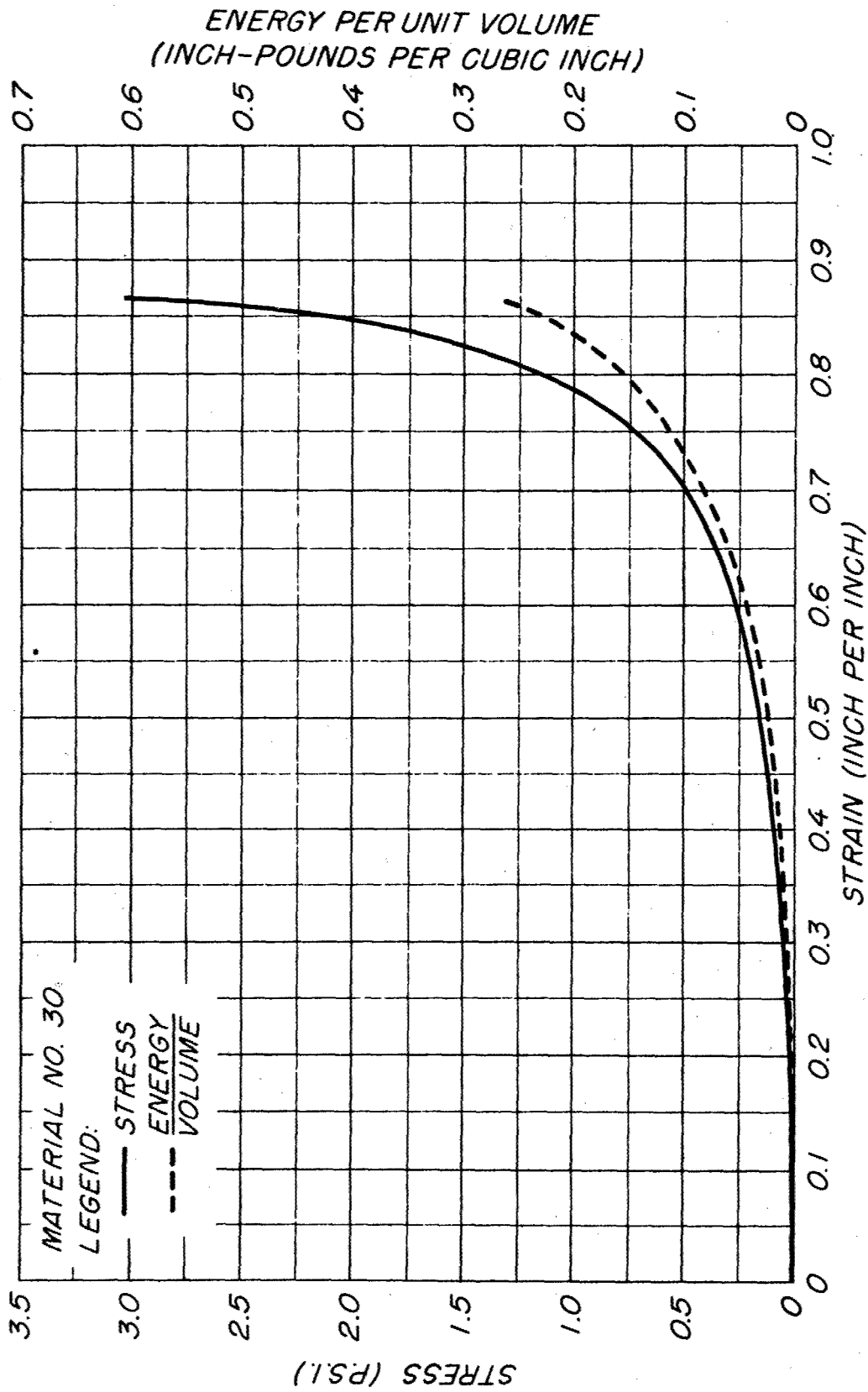


Figure 76. --Stress-strain and energy per unit volume-strain curves for material No. 30
(Stress 0-3.5 P.S.I.).

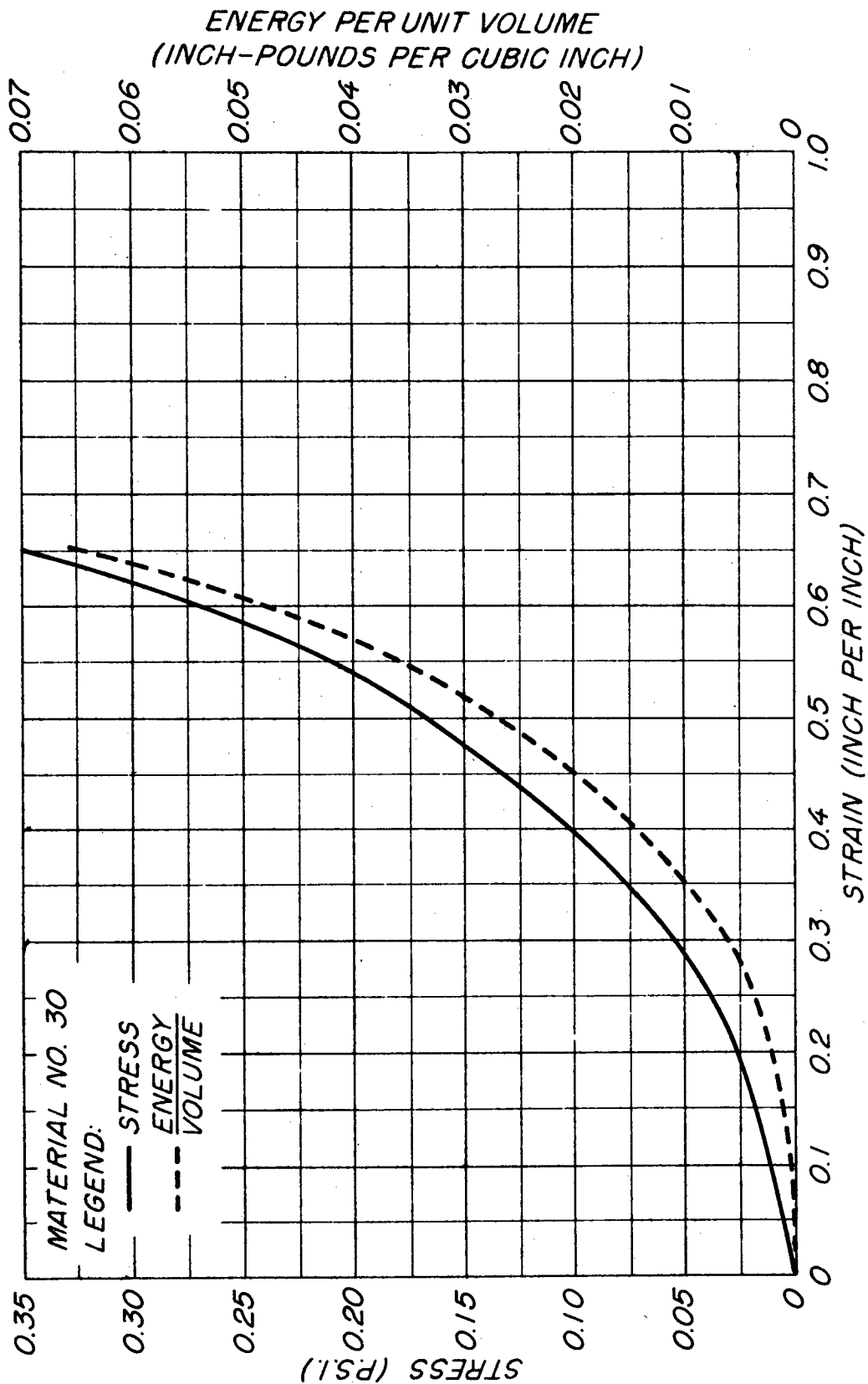


Figure 77.--Stress-strain and energy per unit volume-strain curves for material No. 30
(Stress 0-0.35 P.S.I.).

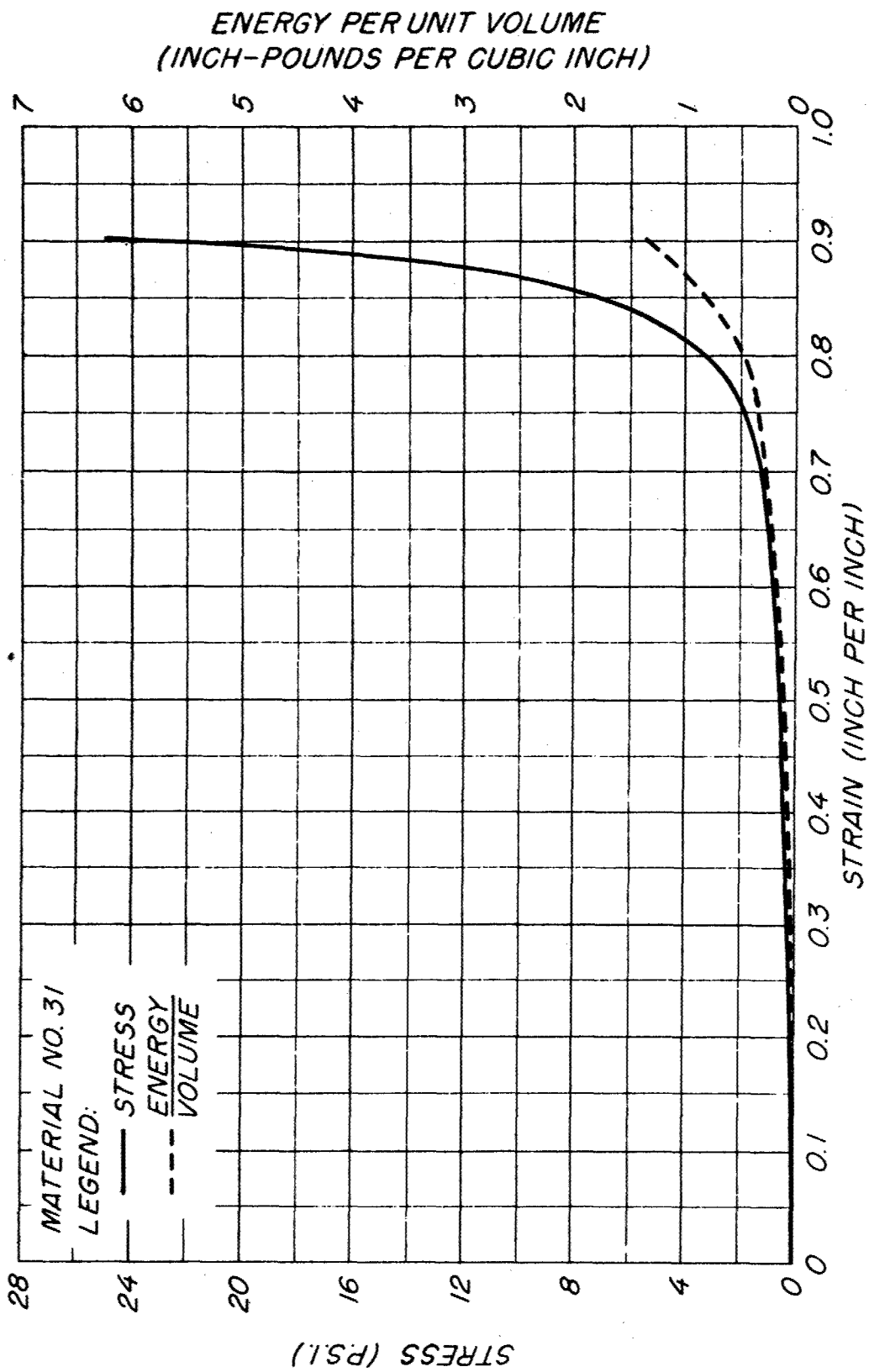


Figure 78.---Stress-strain and energy per unit volume-strain curves for material No. 31
(Stress 0-28 P.S.I.).

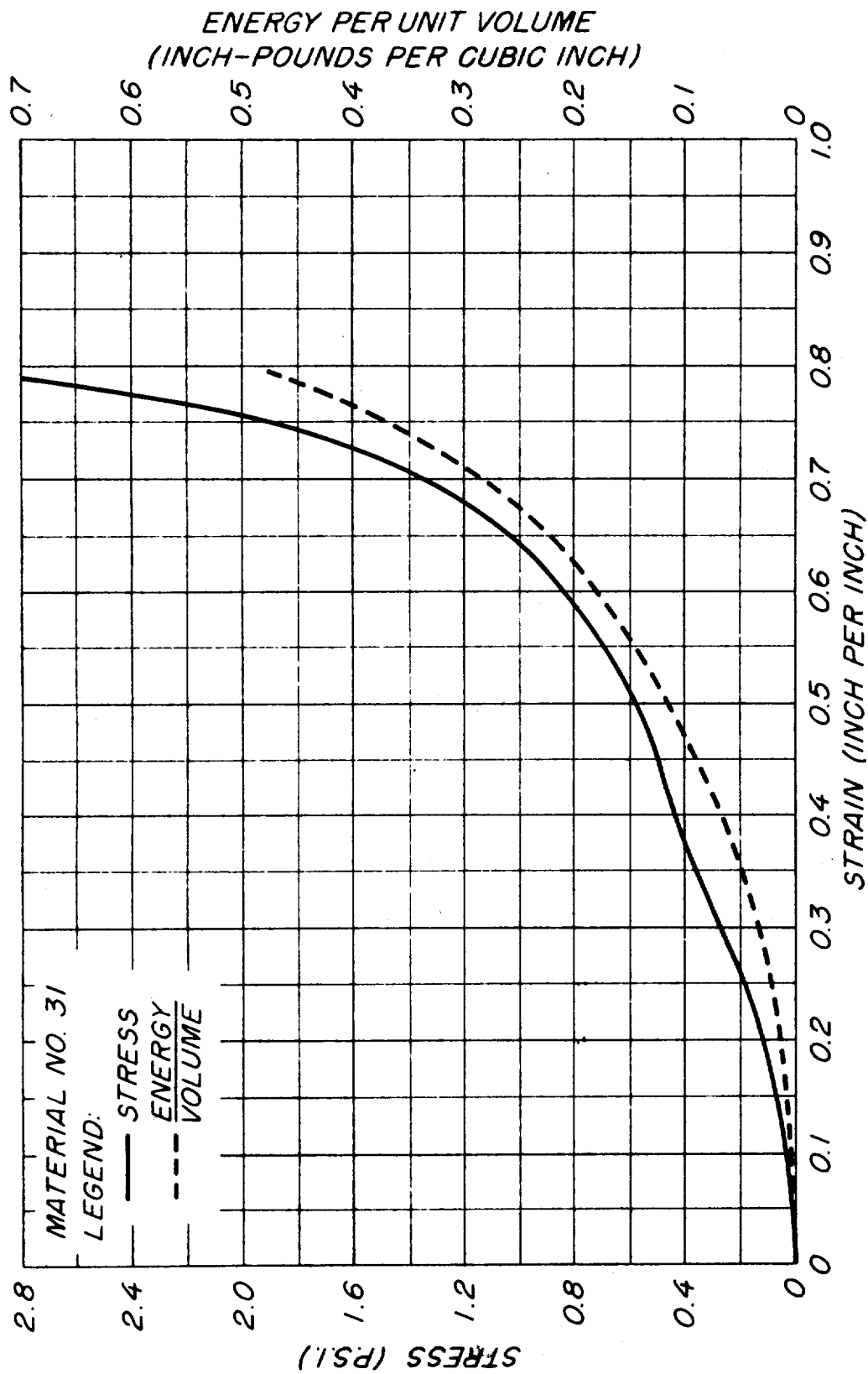


Figure 79.--Stress-strain and energy per unit volume-strain curves for material No. 31
(Stress 0-2.8 P.S.I.).

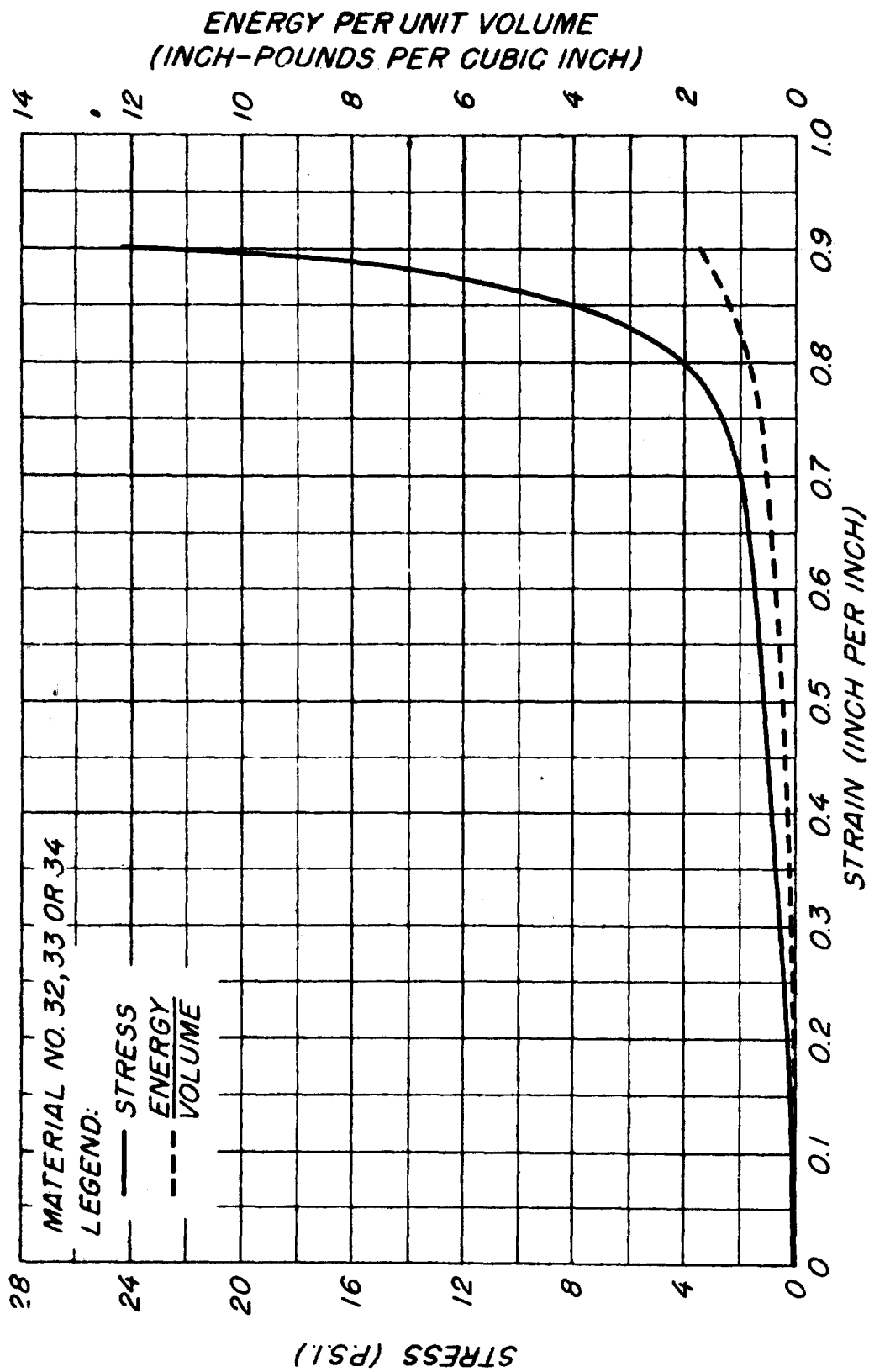


Figure 80.--Stress-strain and energy per unit volume-strain curves for material Nos. 32, 33, or 34 (Stress 0-28 P.S.I.).

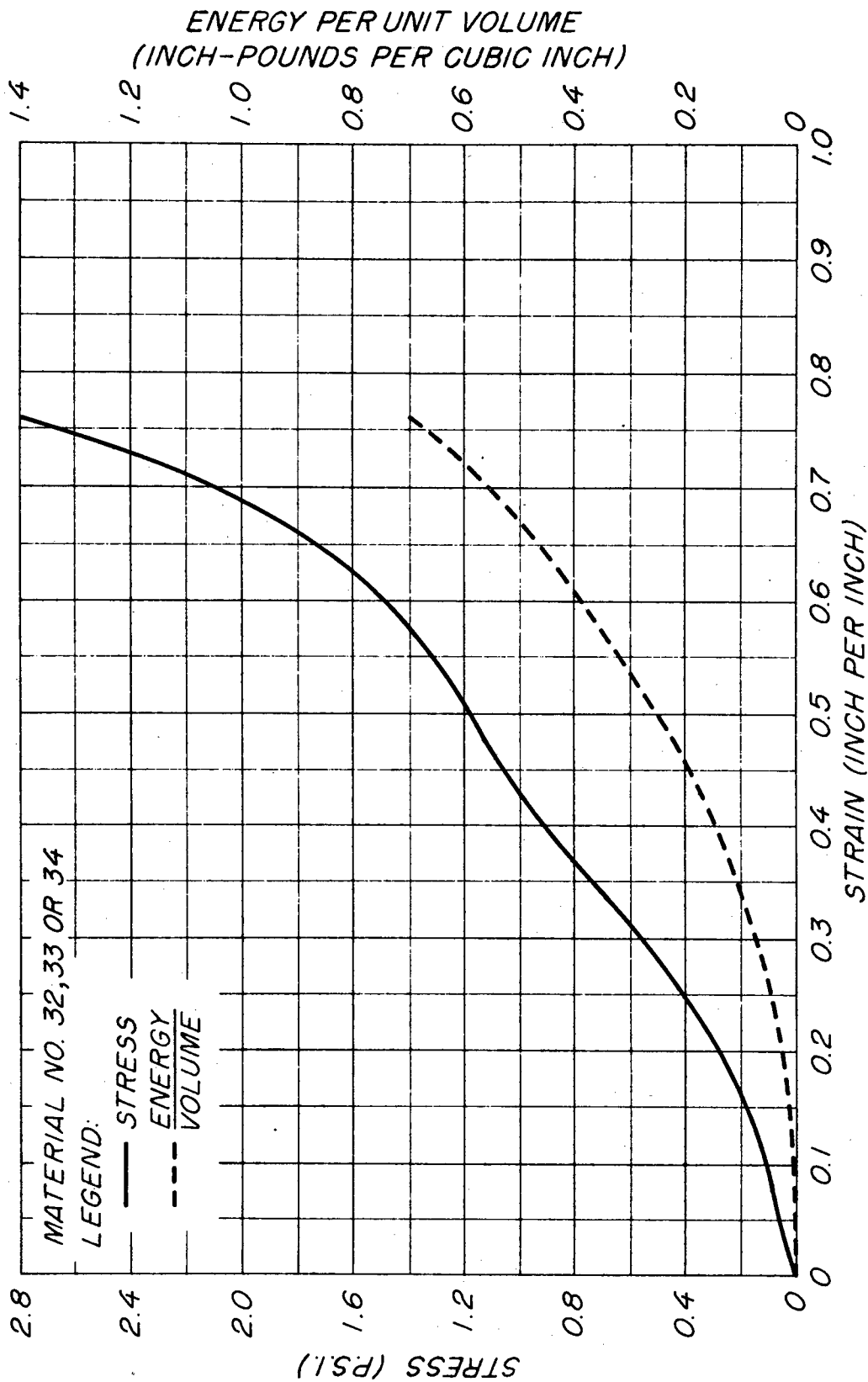


Figure 81.---Stress-strain and energy per unit volume-strain curves for material Nos. 32, 33, or 34 (Stress 0-2.8 P.S.I.).

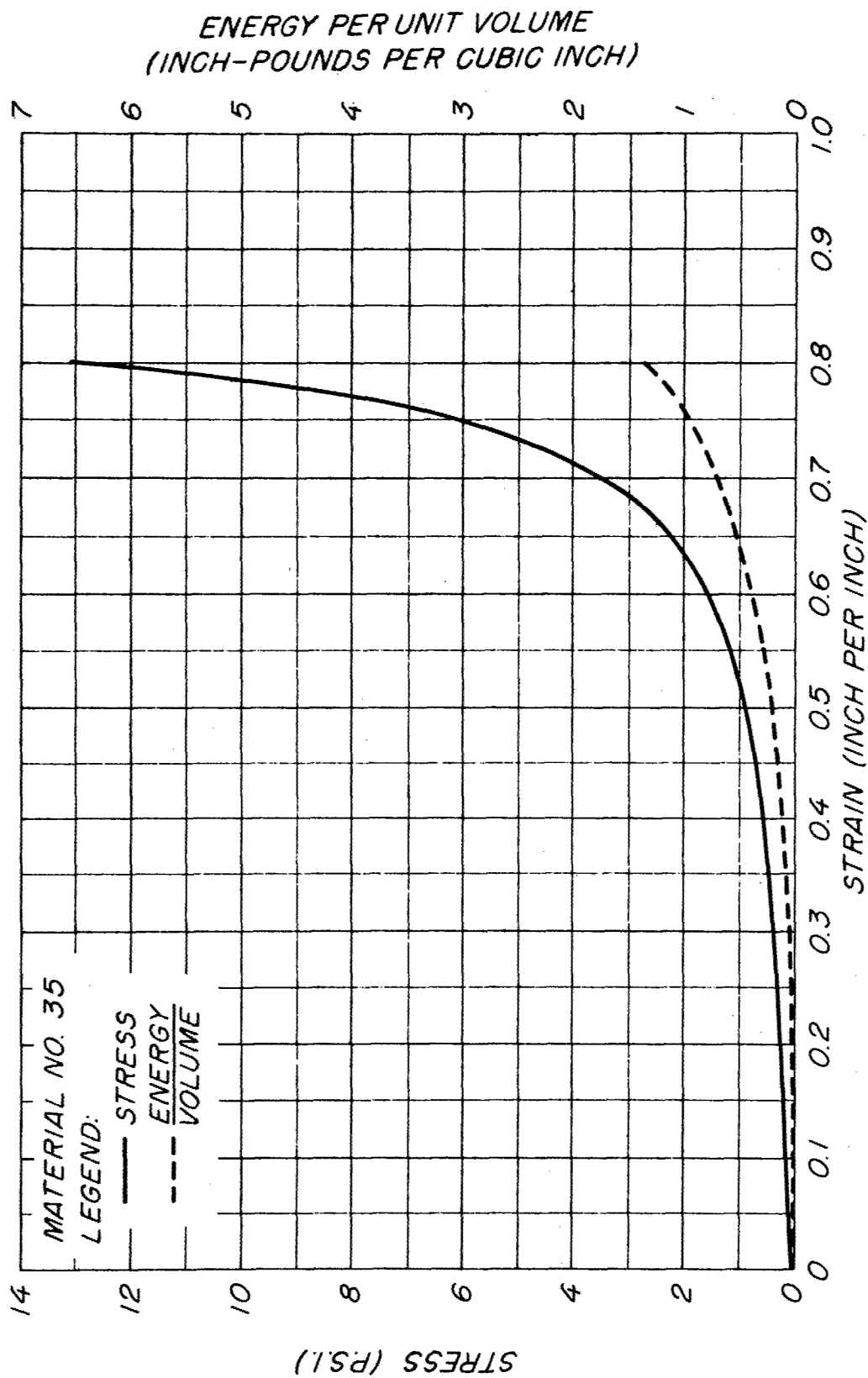


Figure 82.--Stress-strain and energy per unit volume-strain curves for material No. 35.

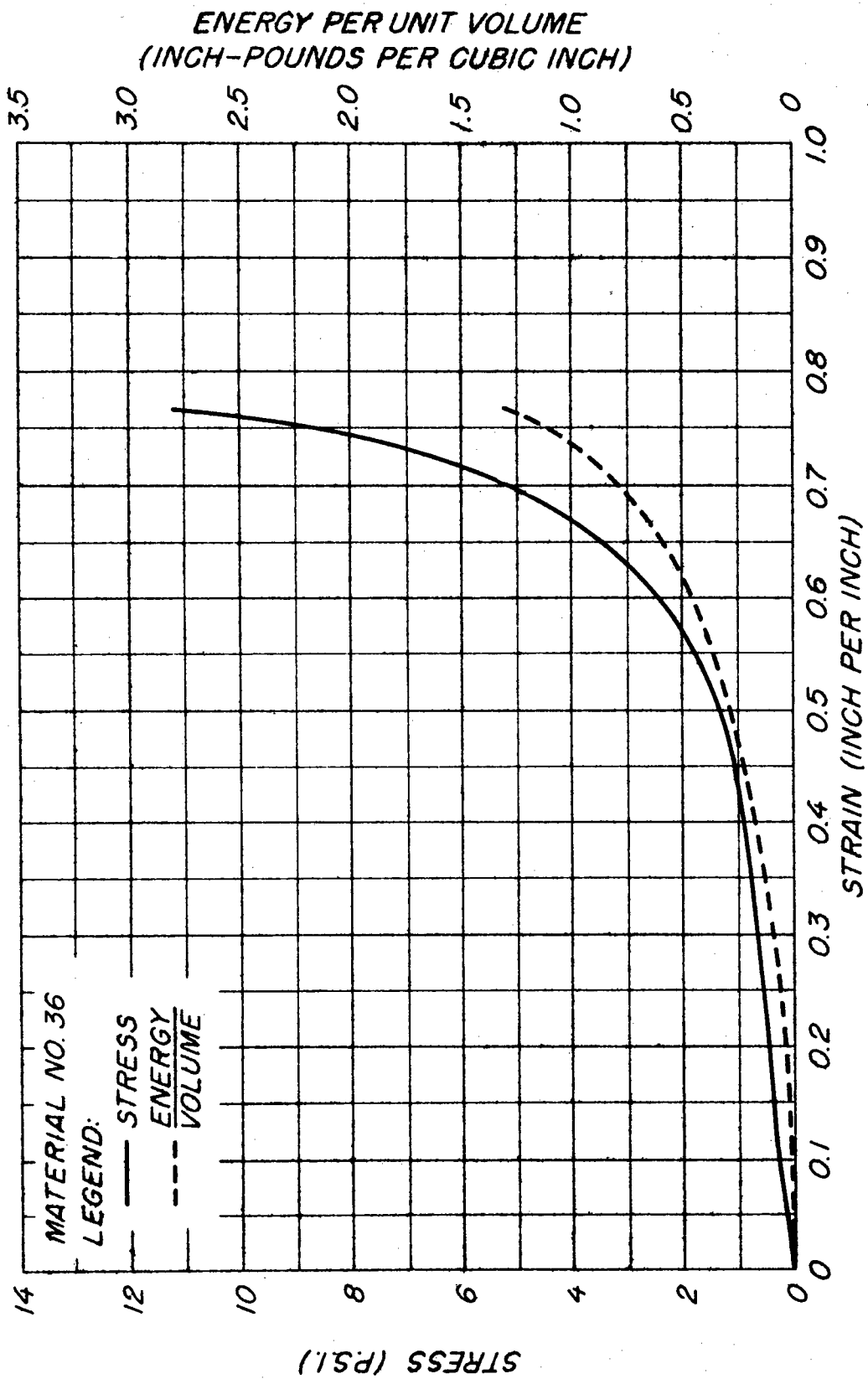


Figure 83.---Stress-strain and energy per unit volume-strain curves for material No. 36.

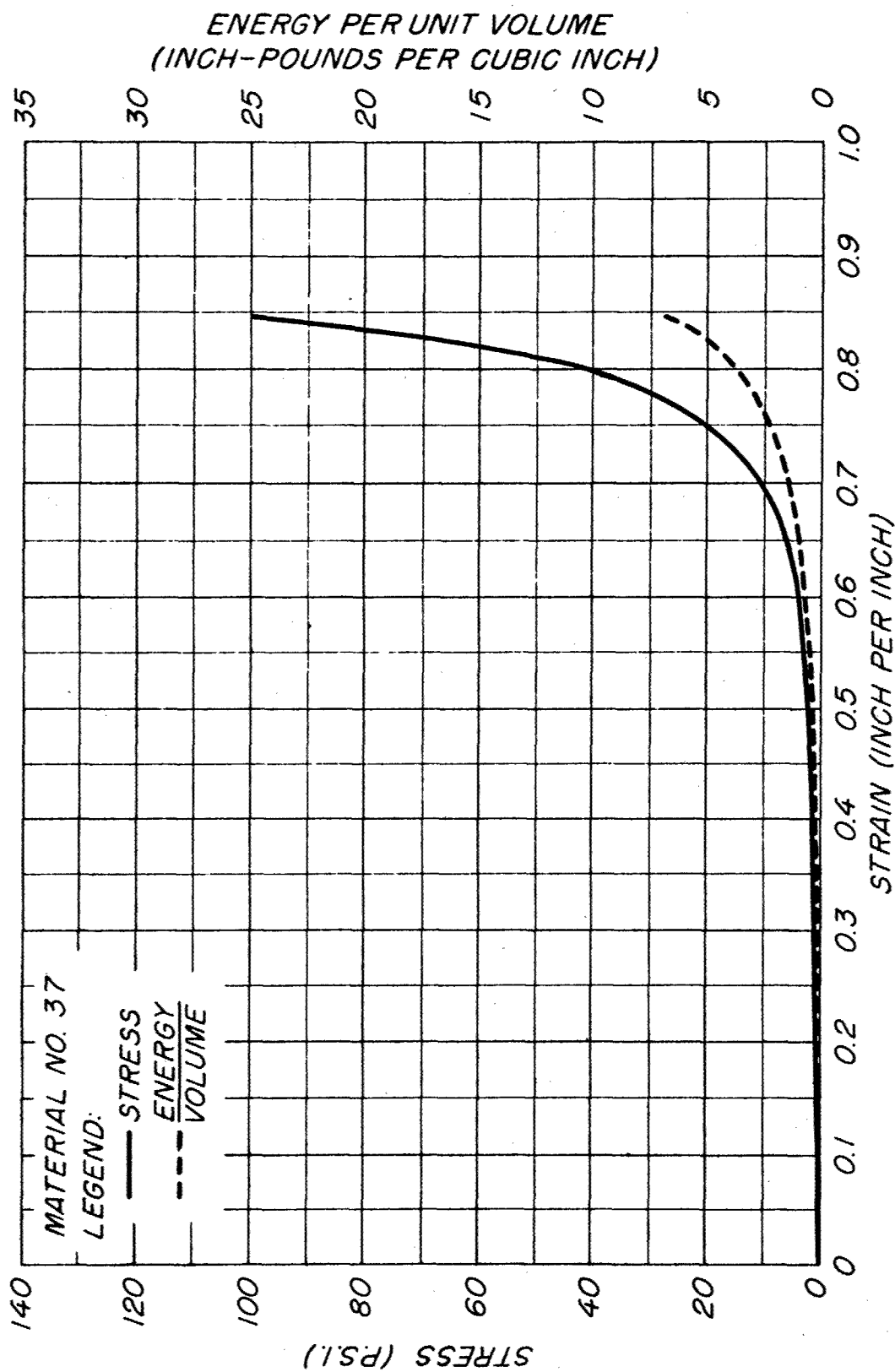


Figure 84.---Stress-strain and energy per unit volume-strain curves for material No. 37
(Stress 0-140 P.S.I.).

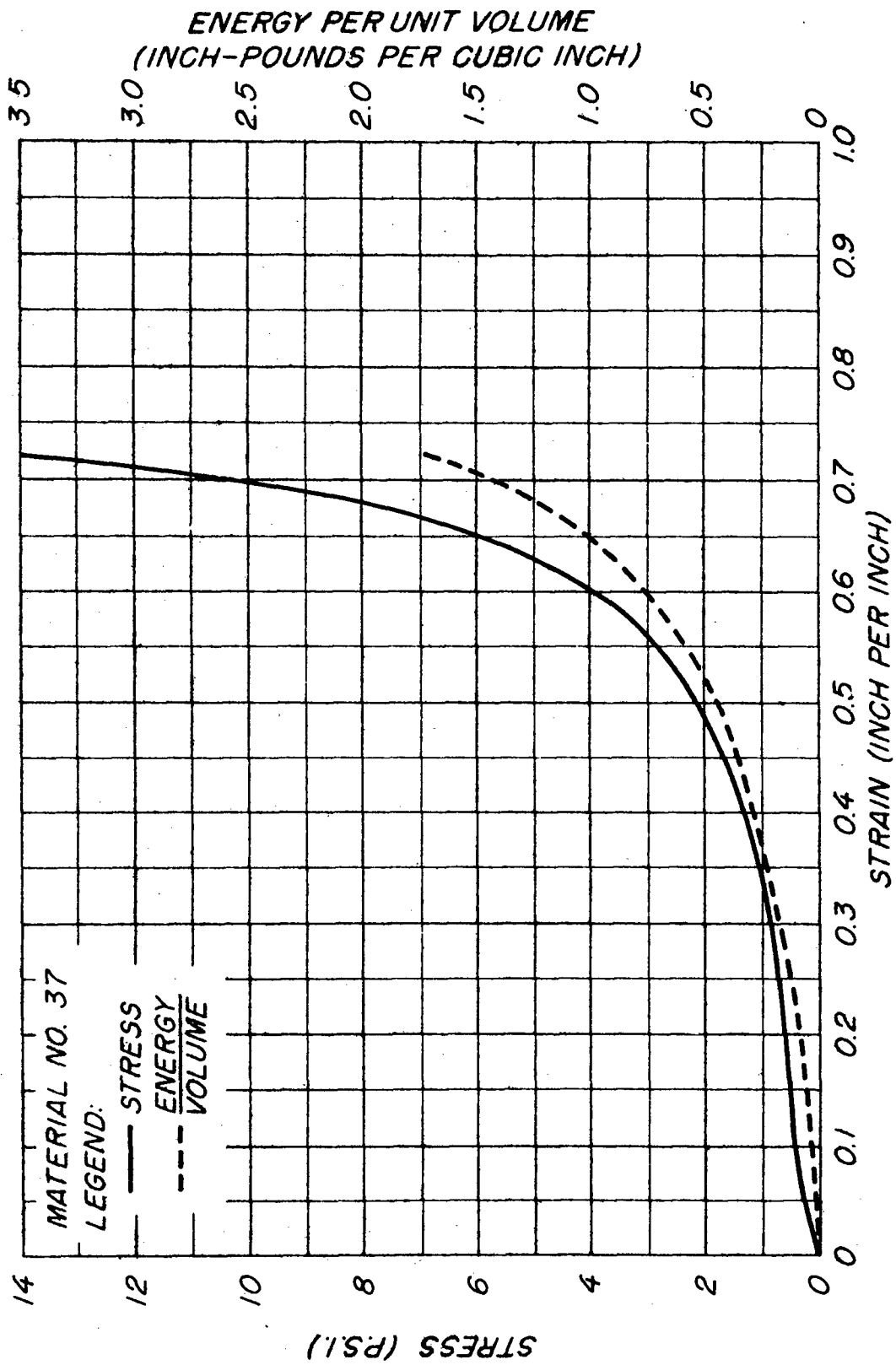


Figure 85.---Stress-strain and energy per unit volume-strain curves for material No. 37
(Stress 0-14 P.S.I.).

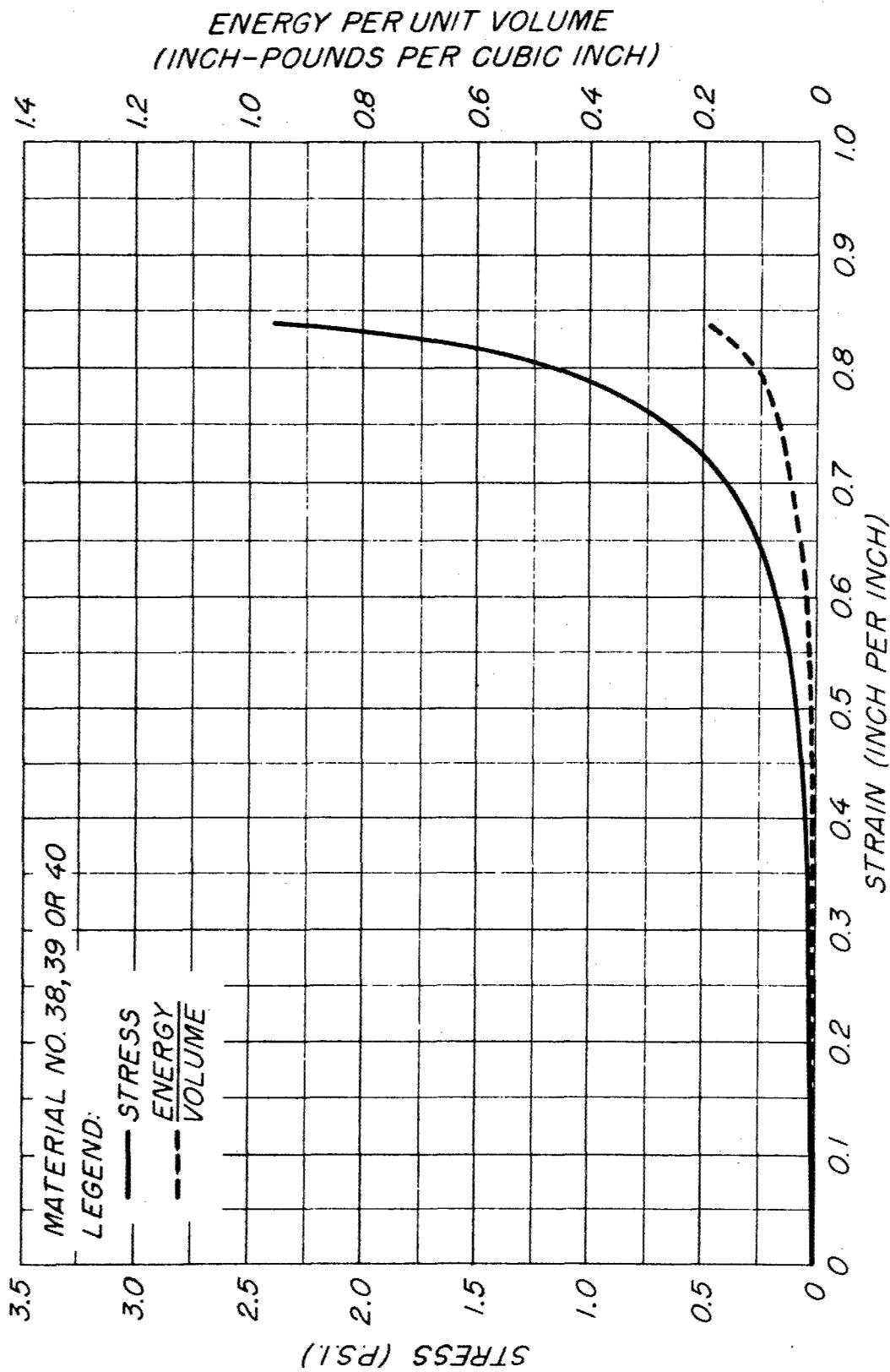


Figure 86.--Stress-strain and energy per unit volume-strain curves for material Nos. 38, 39, or 40 (Stress 0-3.5 P.S.I.).

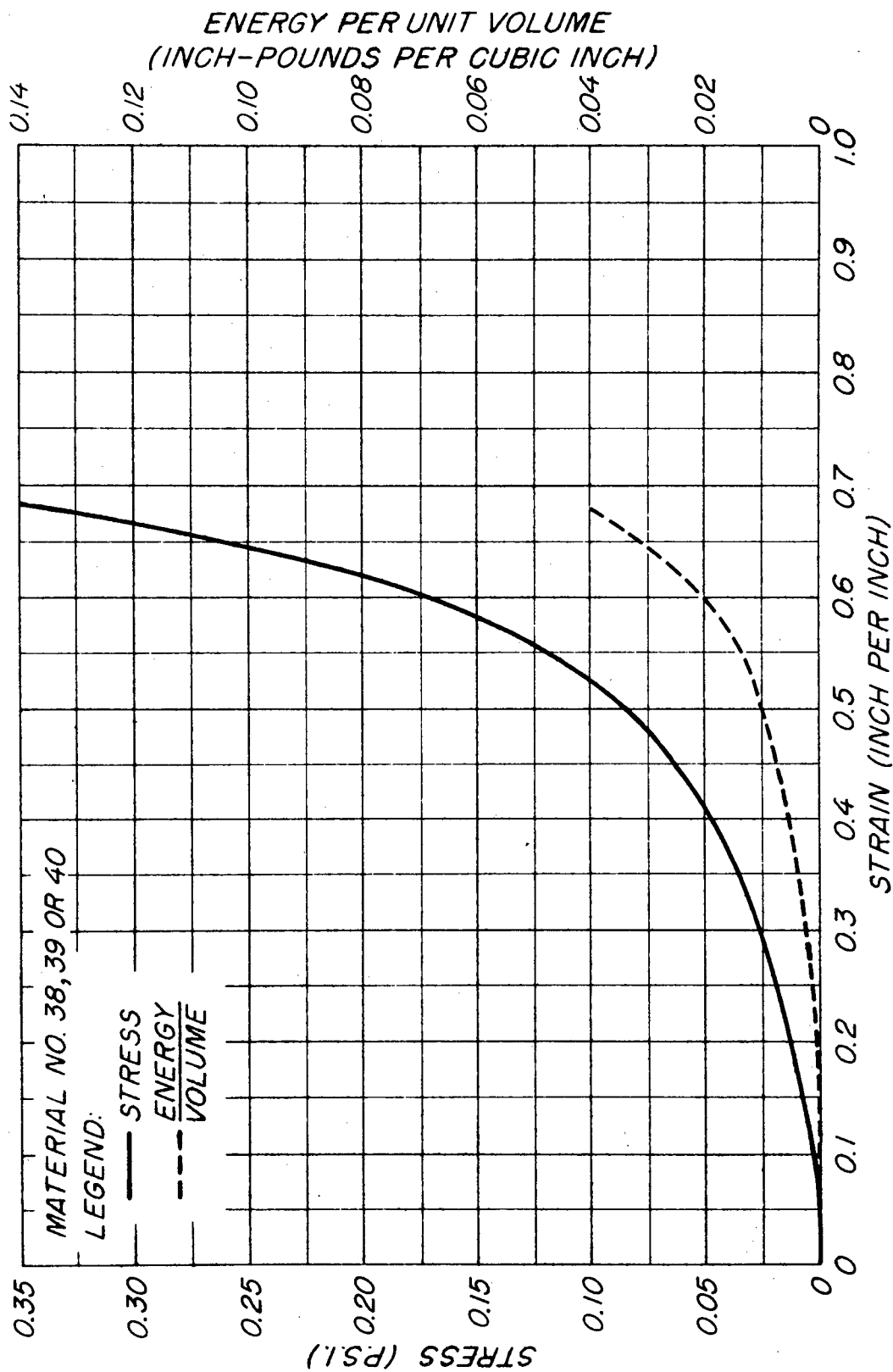


Figure 87.--Stress-strain and energy per unit volume-strain curves for material Nos. 38, 39, or 40 (Stress 0-0.35 P.S.I.).

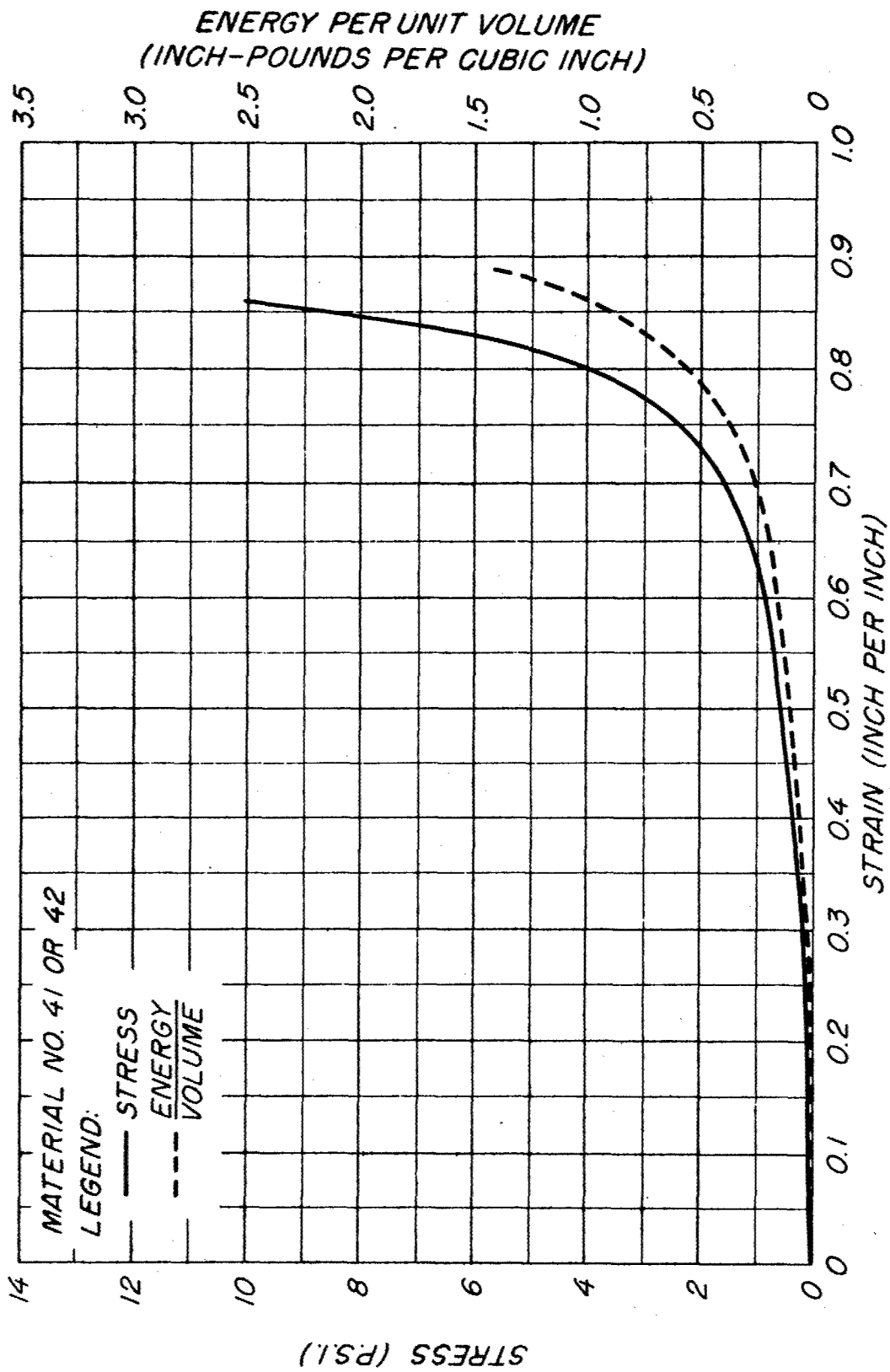


Figure 88.--Stress-strain and energy per unit volume-strain curves for material Nos. 41 or 42 (Stress 0-14 P.S.I.).

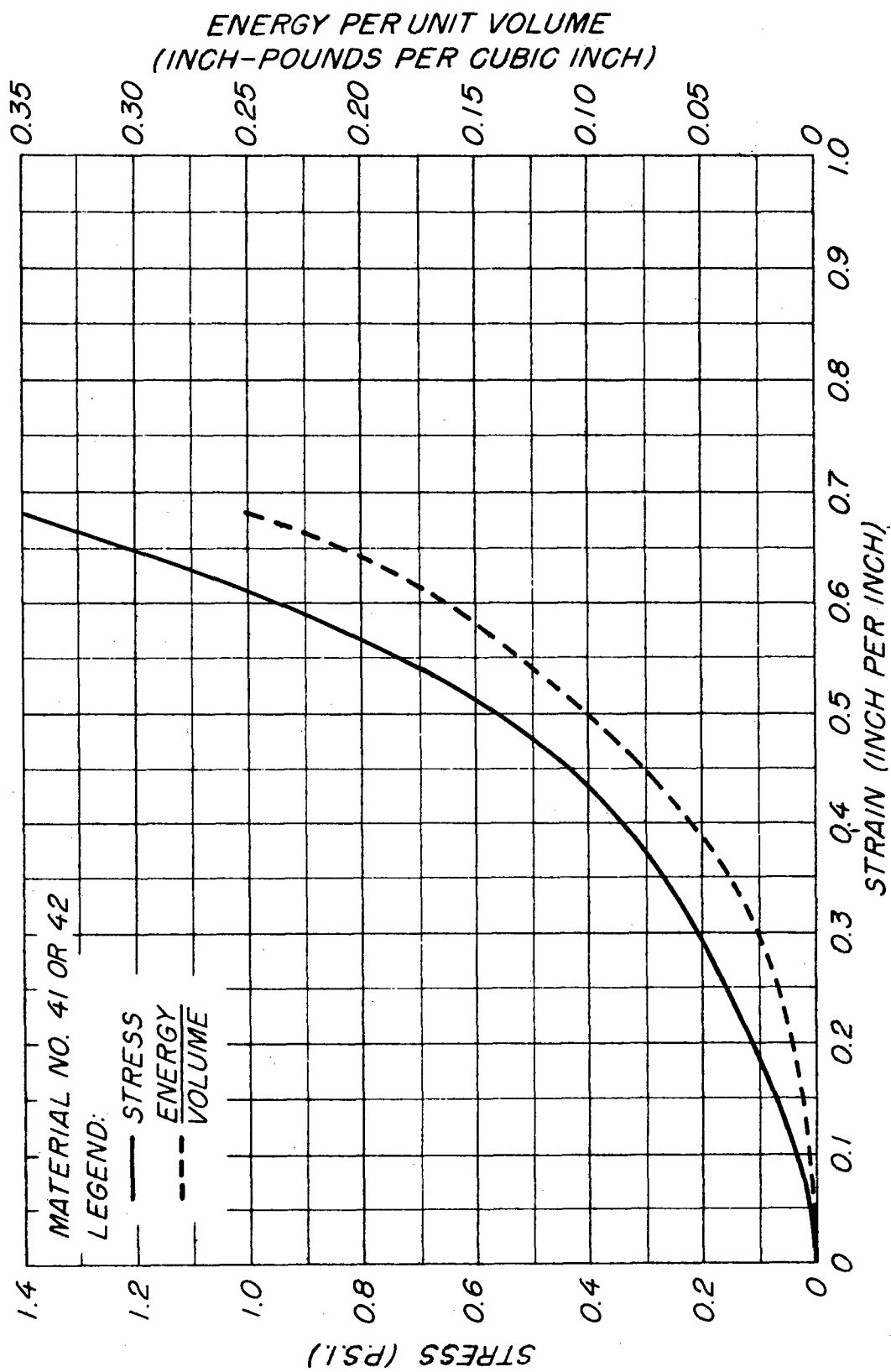


Figure 89.--Stress-strain and energy per unit volume-strain curves for material Nos. 41 or 42 (Stress 0-1.4 P.S.I.).

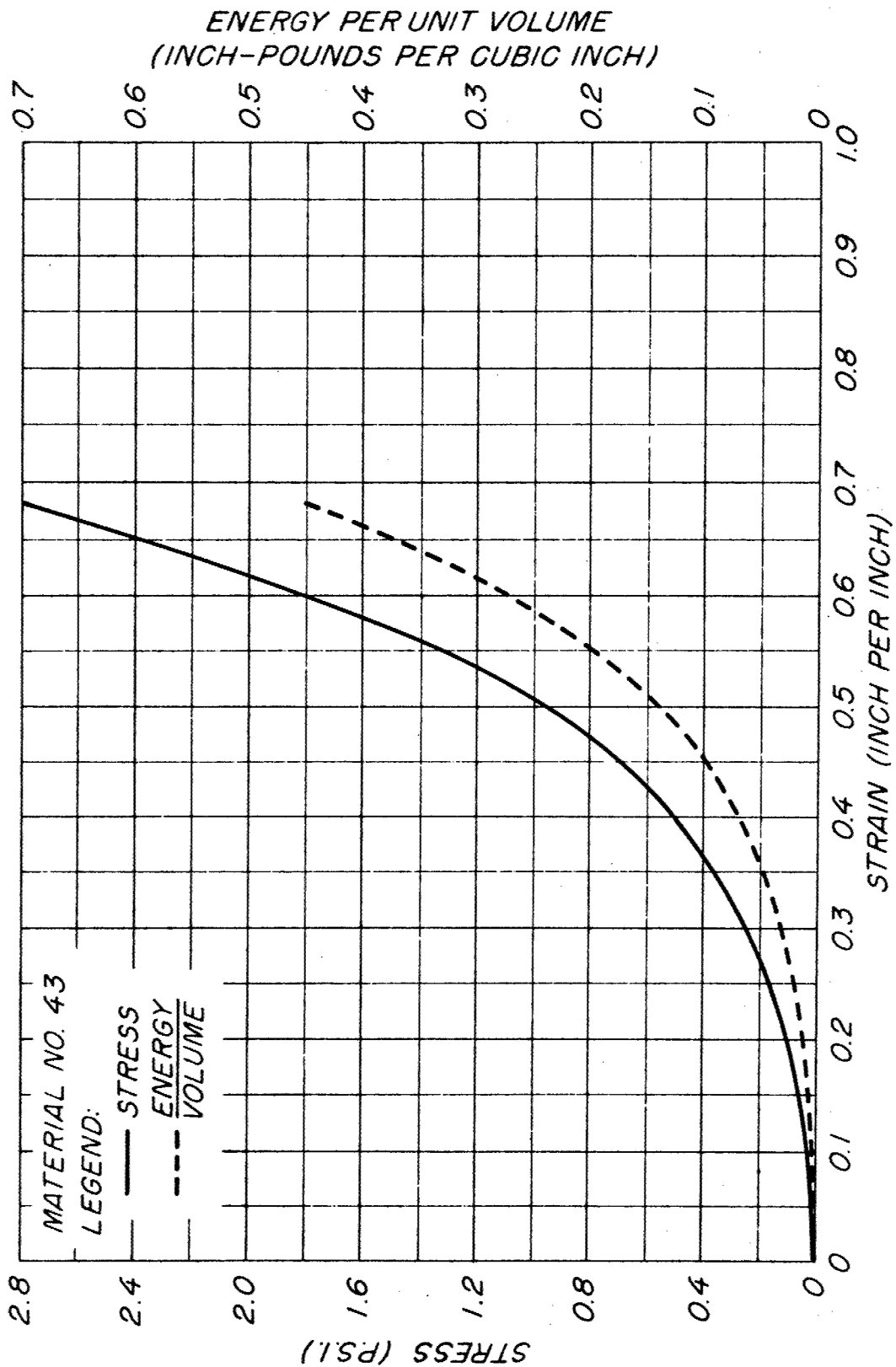


Figure 90.--Stress-strain and energy per unit volume-strain curves for material No. 43
(Stress 0-2.8 P.S.I.).

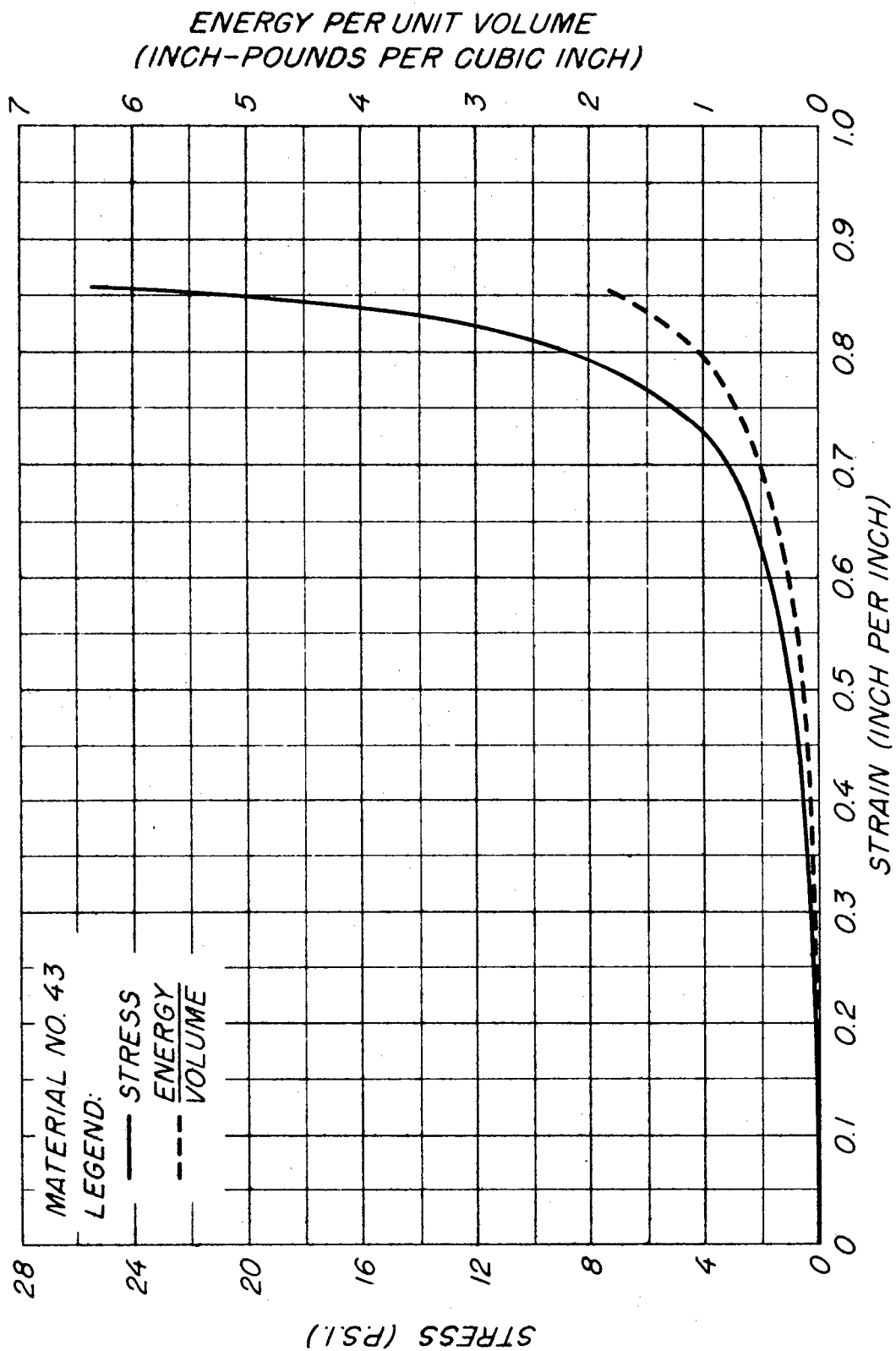


Figure 91.---Stress-strain and energy per unit volume-strain curves for material No. 43
(Stress 0-28 P.S.I.).

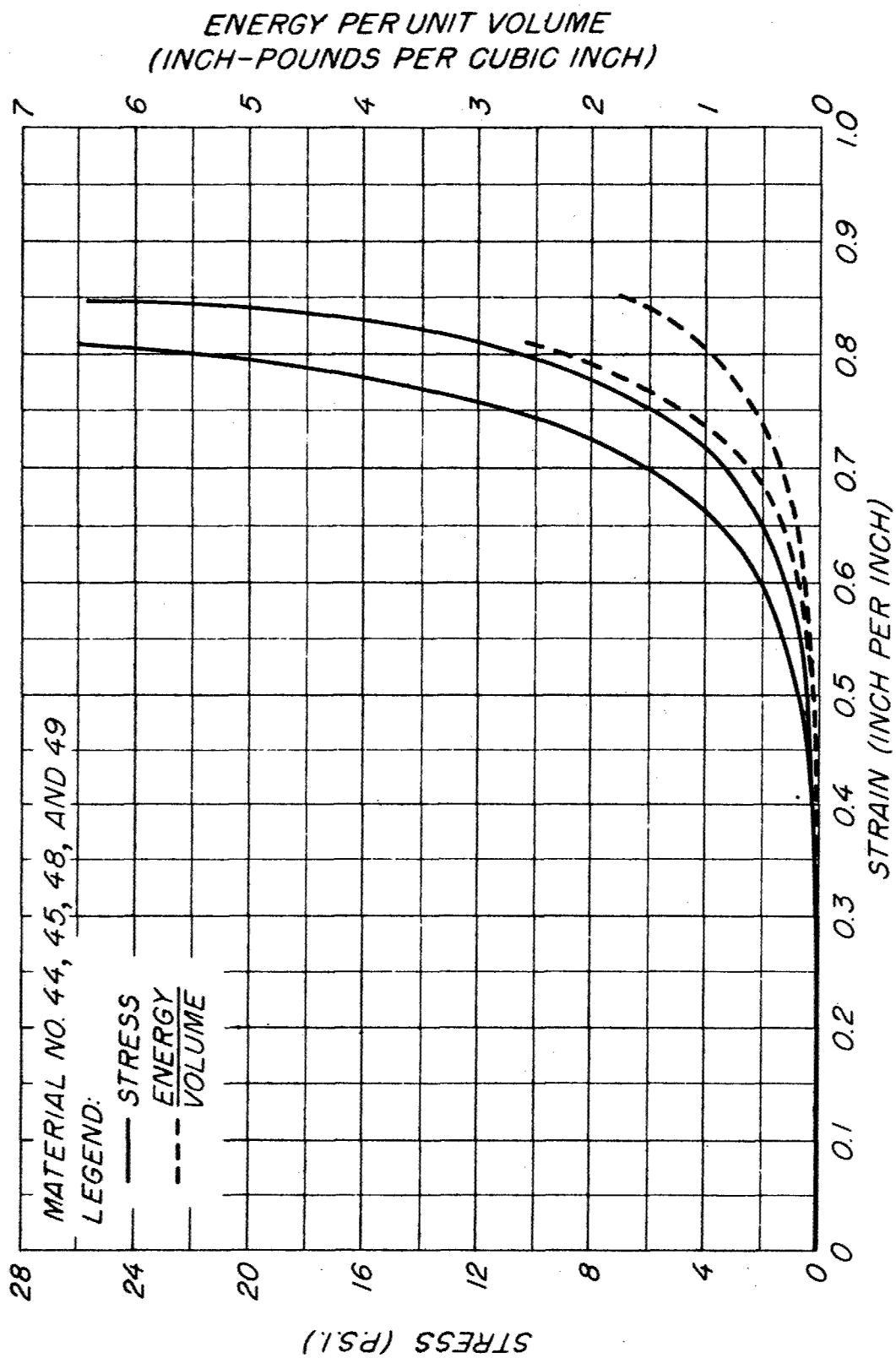


Figure 92.---Stress-strain and energy per unit volume-strain curves for material Nos. 44, 45, 48, and 49 (Stress 0-28 P.S.I.).

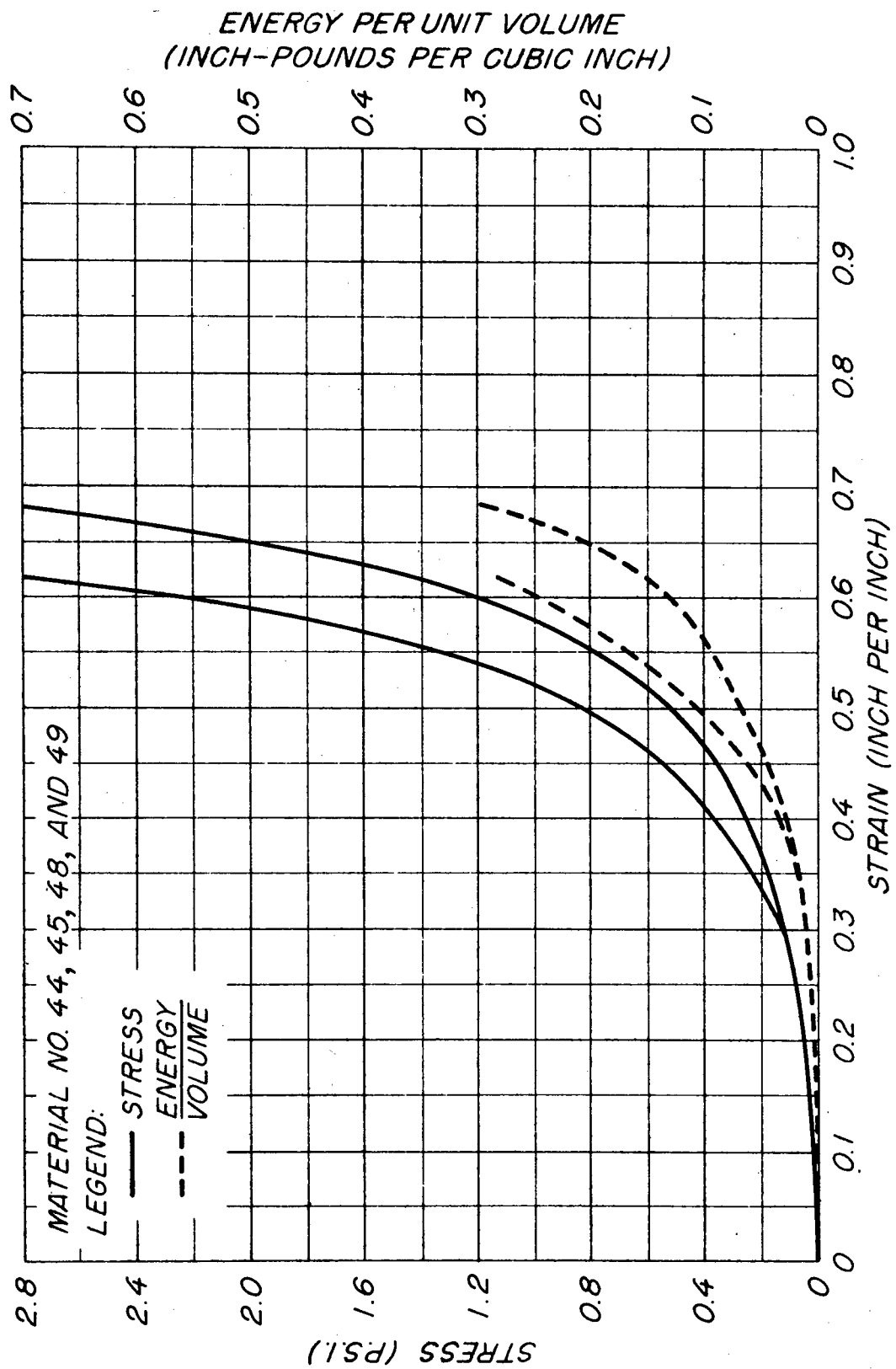


Figure 93.---Stress-strain and energy per unit volume-strain curves for material Nos. 44, 45, 48, or 49 (Stress 0-2.8 P.S.I.).

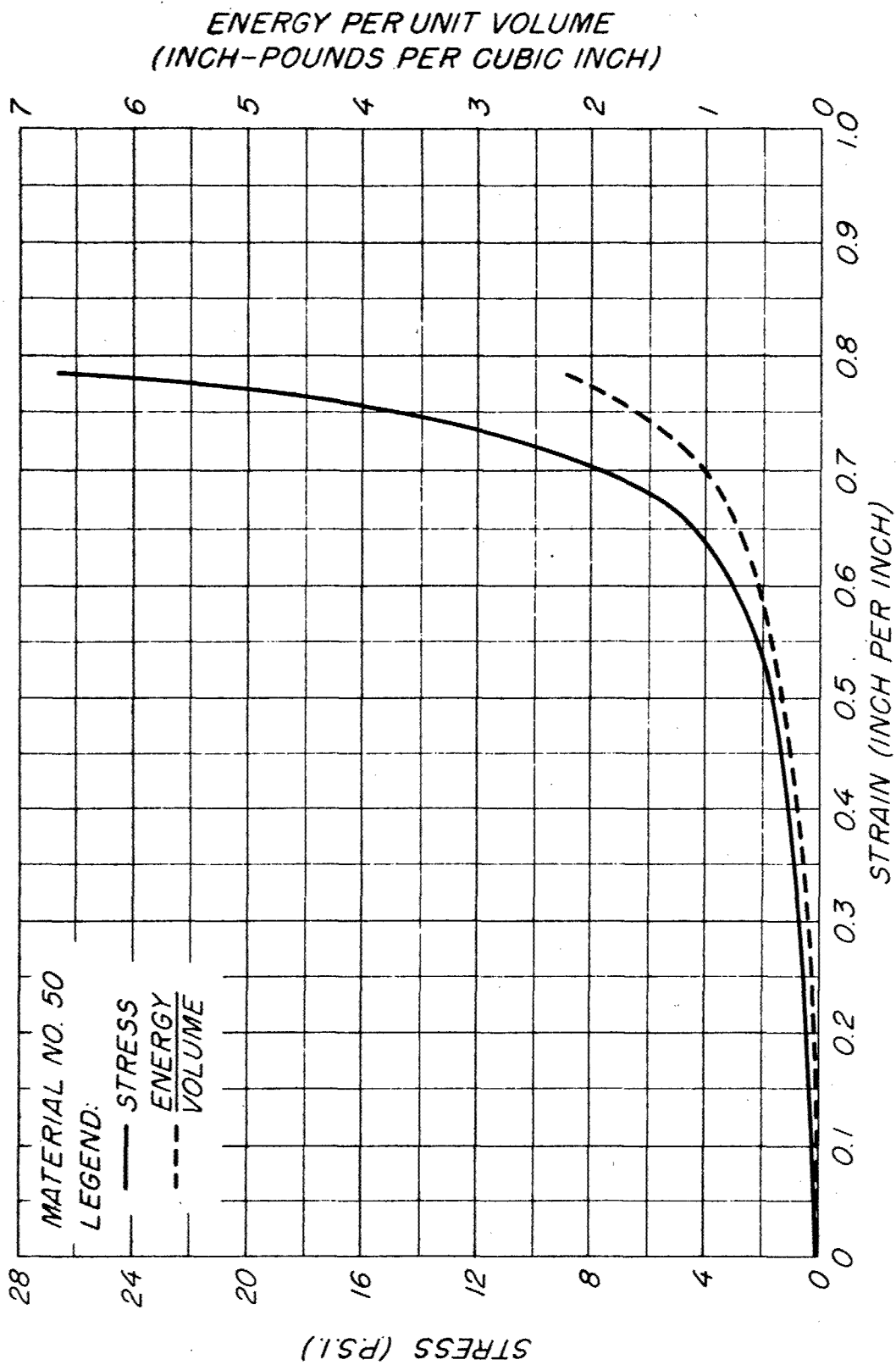


Figure 94.---Stress-strain and energy per unit volume-strain curves for material No. 50
(Stress 0-28 P.S.I.).

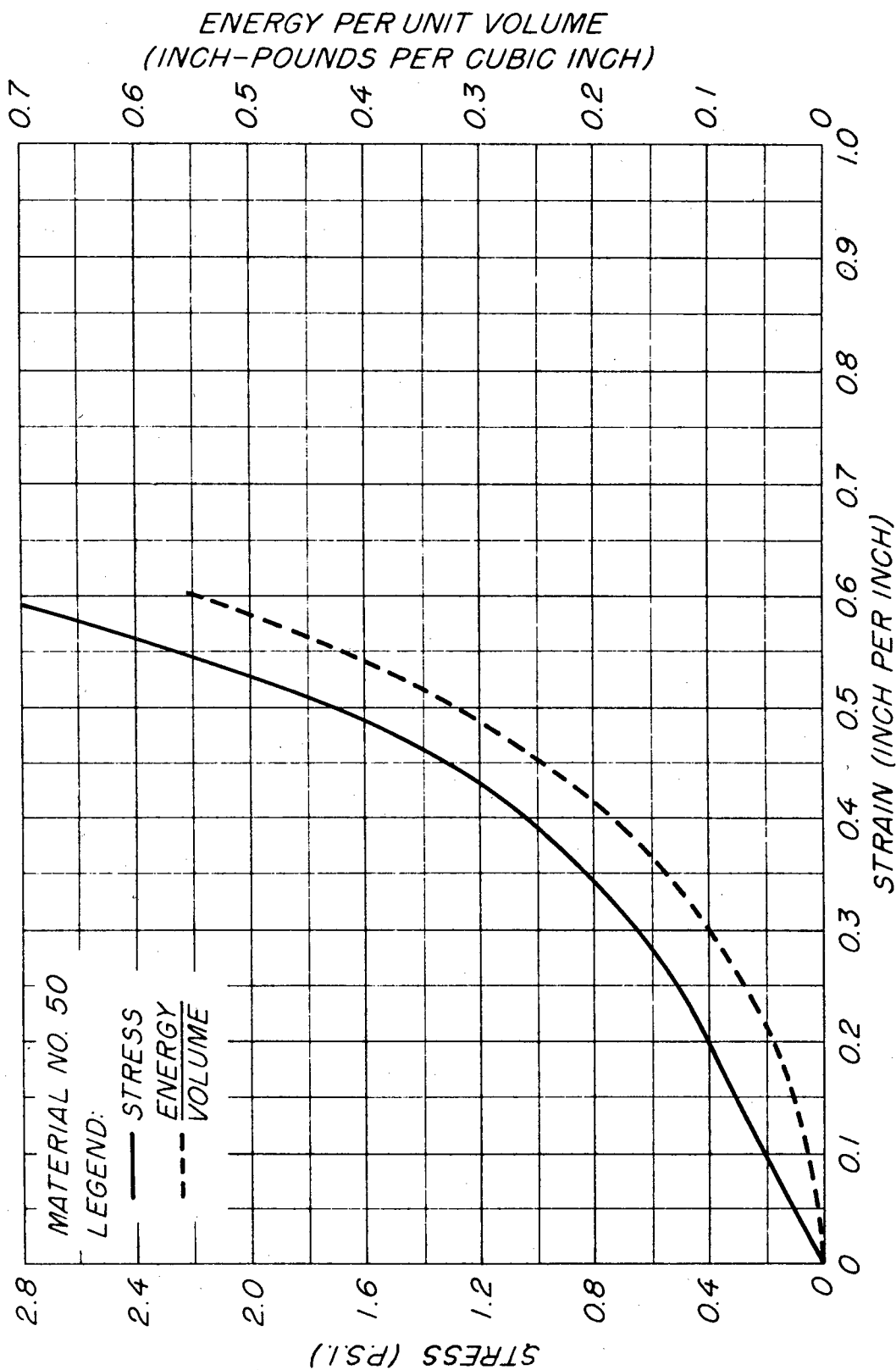


Figure 95.--Stress-strain and energy per unit volume-strain curves for material No. 50
(Stress 0-2.8 P.S.I.).

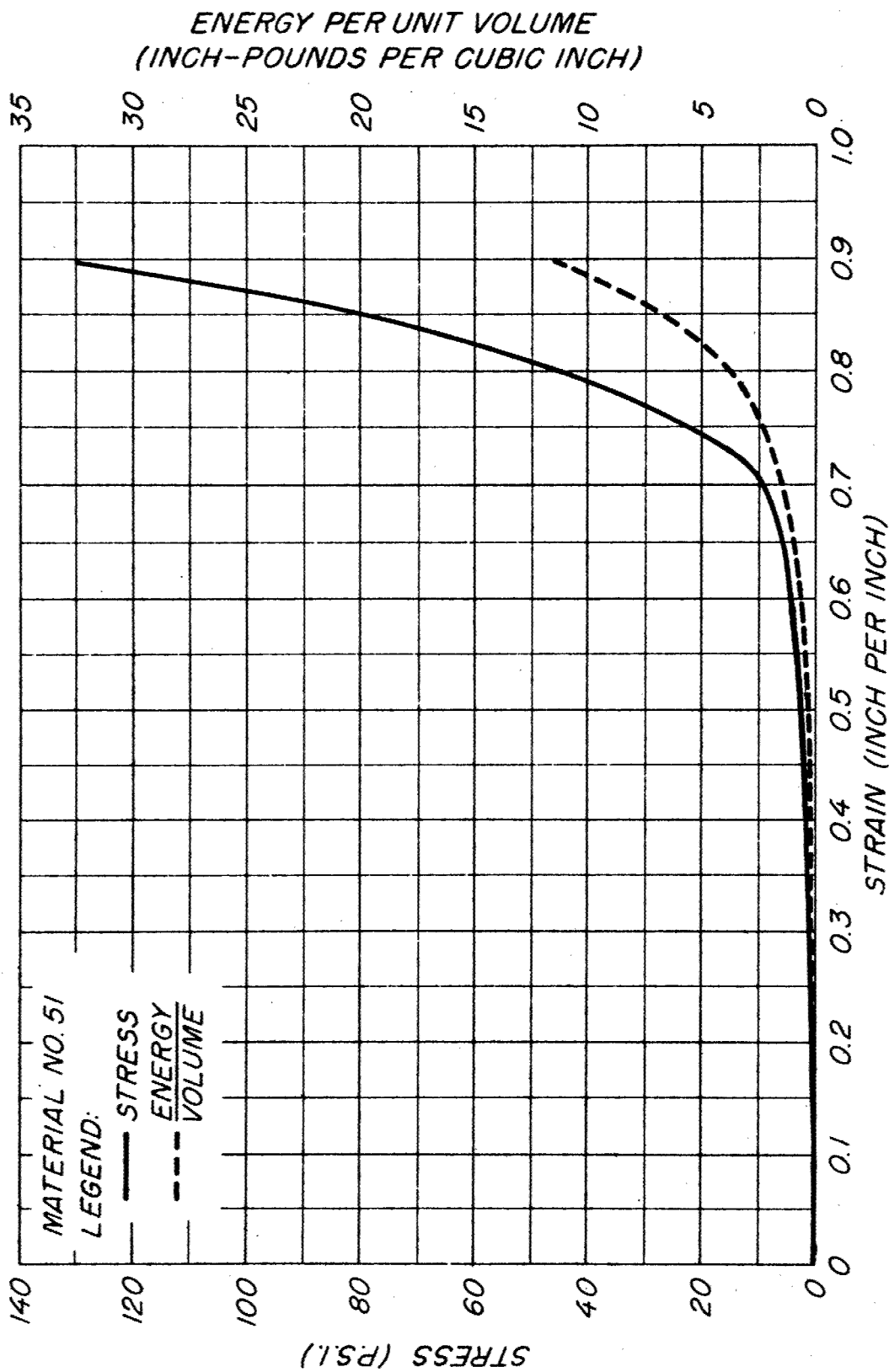


Figure 96.---Stress-strain and energy per unit volume-strain curves for material No. 51
(Stress 0-140 P.S.I.).

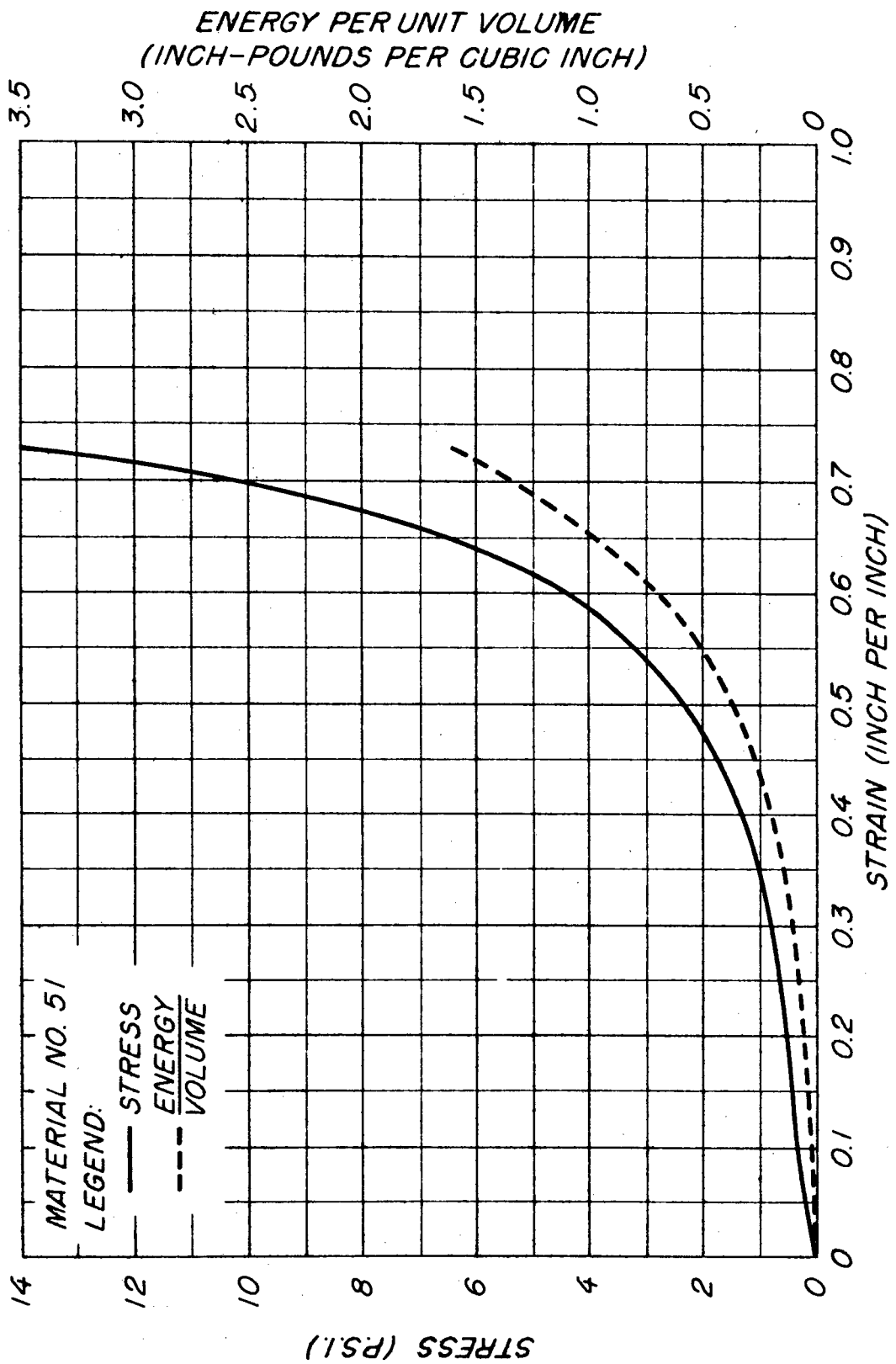


Figure 97.--Stress-strain and energy per unit volume-strain curves for material No. 51
(Stress 0-14 P.S.I.).

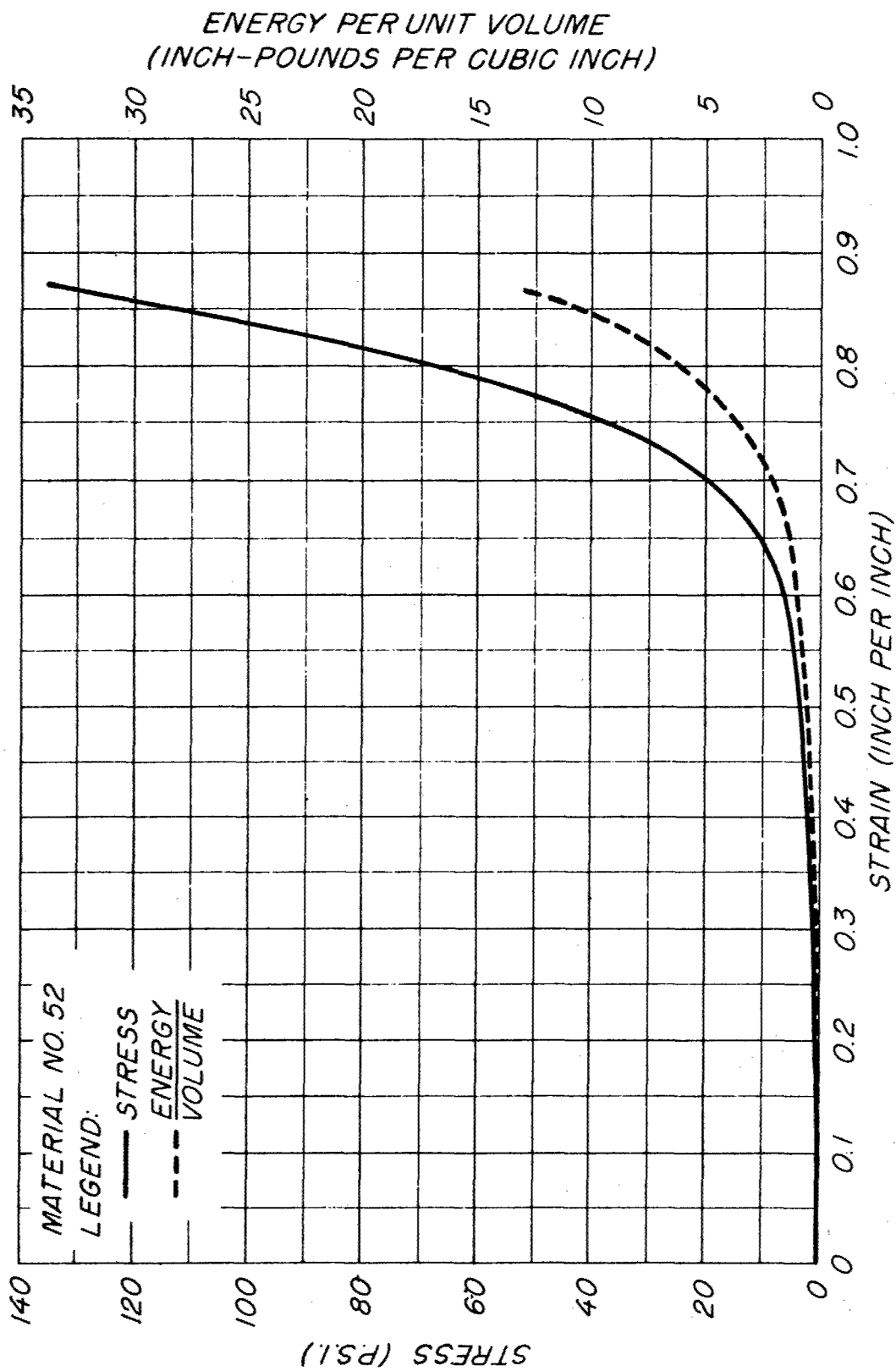


Figure 98.--Stress-strain and energy per unit volume-strain curves for material No. 52
 (Stress 0-140 P.S.I.).

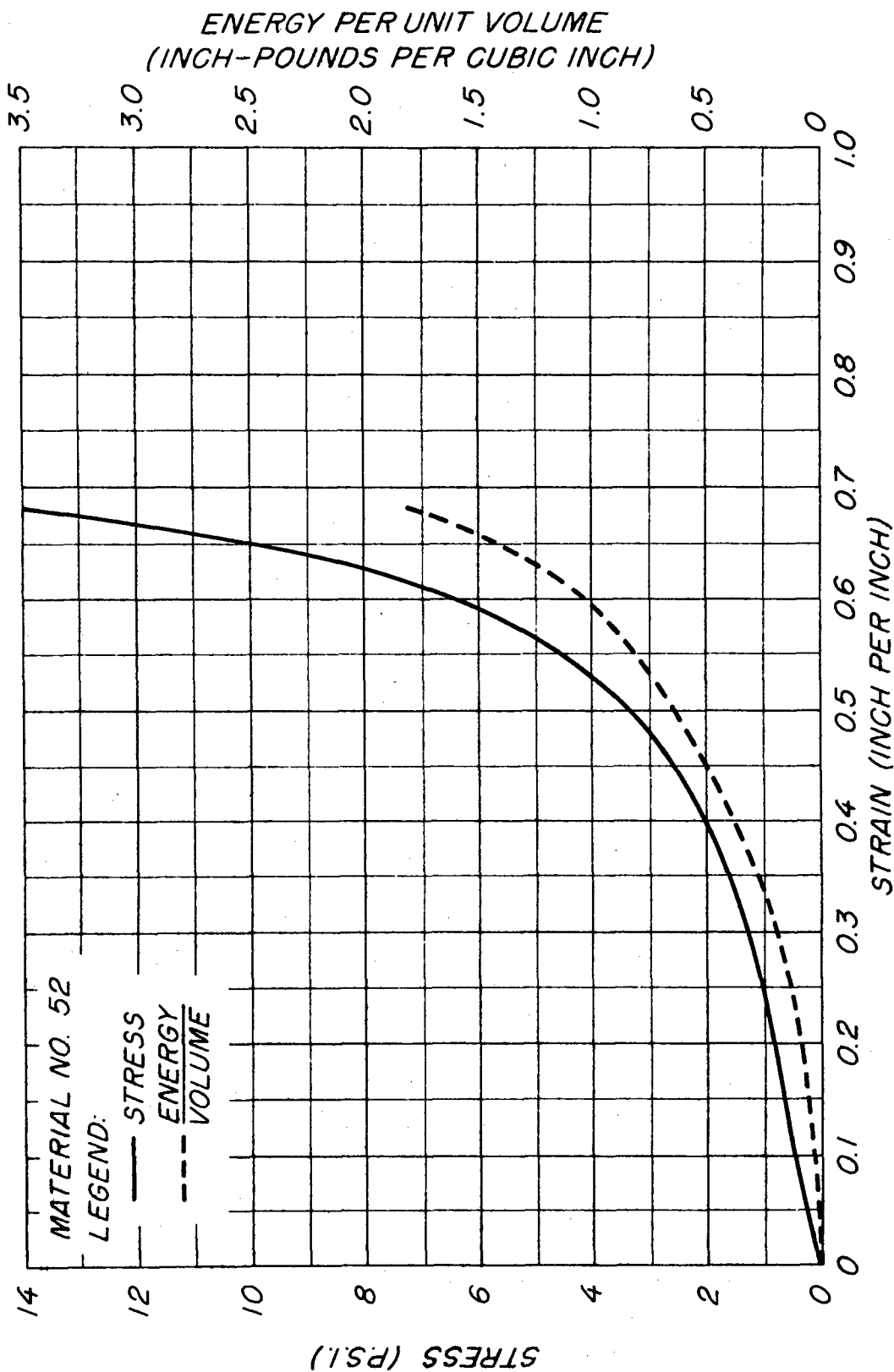


Figure 99.--Stress-strain and energy per unit volume-strain curves for material No. 52
(Stress 0-14 P.S.I.).

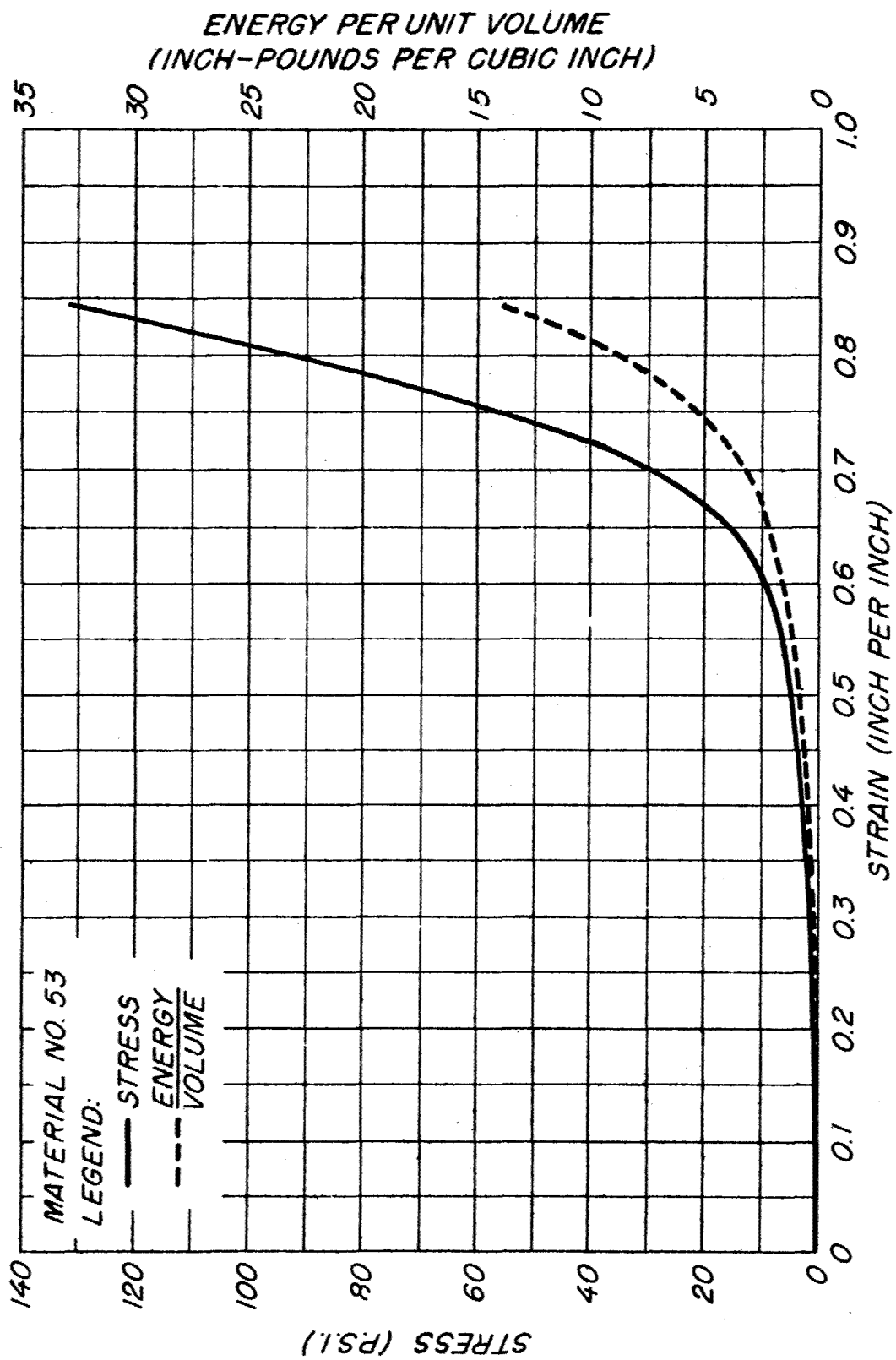


Figure 100. ---Stress-strain and energy per unit volume-strain curves for material No. 53
(Stress 0-140 P.S.I.).

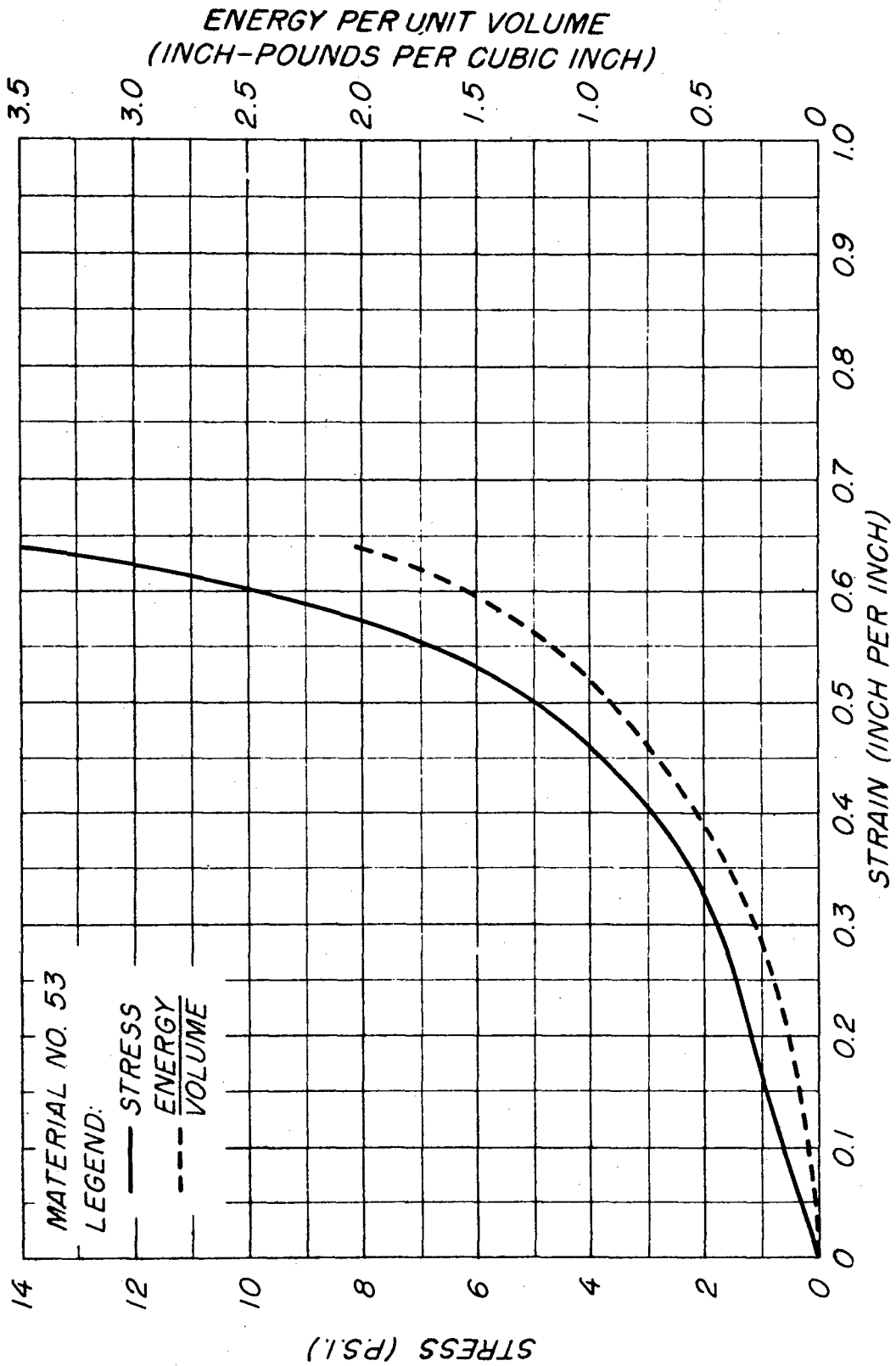


Figure 101.--Stress-strain and energy per unit volume-strain curves for material No. 53
(Stress 0-14 P.S.I.).